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Динамика кварков в метрике адронов и структура барионов

Dynamics of quarks in the hadrons metrics with application to the baryon structure

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В работе рассмотрена система уравнений Дирака, описывающая динамику кварков в метрике адронов. Предполагается, что взаимодействие кварков осуществляется через поле Янга-Миллса и электромагнитное поле. Сформулирована замкнутая модель барионов в случае стационарной метрики. Вычислены магнитные моменты протона, нейтрона и лямбда бариона.

Ключевые слова: адроны, кварки, магнитный момент, метрика, лямбда барион, нейтрон, протон, уравнение Дирака, теория Янга-Миллса.

In this paper we consider a system of Dirac equations describing the dynamics of quarks in hadrons metric. We assume that the quarks interact via an external Yang-Mills field. Under these assumptions, we formulated a closed model of the baryons in a stationary metric. Magnetic moments of the proton, neutron and lambda baryon have been calculated.

Keywords: hadrons, quarks, magnetic moment, metric, lambda baryon, neutron, proton, Dirac equation, Yang-Mills equations.

Introduction

In certain models, lattice quantum chromodynamics (LQCD) is used mainly flat metric [1-6]. In [7] formulated a model of hadrons metric satisfying the basic requirements of particle physics and cosmology. In this paper the dynamics of quarks in the metric [7] has been simulated. The results obtained on the magnetic moments of baryons can explain some of the paradoxes of the theory of quarks.

The basic equations of the model metrics hadrons

In [8] had all the solutions of the Yang-Mills equations in the case of a centrally symmetric metric. A particular case of the centrally symmetric metric is

$$\Psi = \eta_{ij}\omega^i\omega^j = -dt^2 + e^{2\nu}dr^2 + d\theta^2 + \sigma^2(\theta)d\varphi^2$$
$$\frac{d^2\sigma}{d\theta^2} = -\kappa\sigma \tag{1}$$
$$\omega^1 = dt, \omega^2 = e^{\nu}dr, \omega^3 = d\theta, \omega^4 = \sigma d\varphi$$

Here $\eta_{ij} = \eta^{ij}$ is the metric tensor of the Minkowski space of signature (- + + +), $\kappa = const$ is Gaussian curvature of the quadratic form $d\theta^2 + \sigma^2(\theta)d\varphi^2$, function $\nu = \nu(r,t)$ is determined by solving the Yang-Mills equations. Wherever not specified, the system of units in which $\hbar = c = 1$.

Among all the solutions of the Yang-Mills theory, obtained in [8] in the case of the metric (1), there is, which is expressed in terms of Weierstrass elliptic function. In this case, the model equations have the form:

$$A_{\tau\tau} = \frac{1}{2}(A^2 - \kappa^2), e^{\nu} = A_\tau, \quad \tau = t \pm r + \tau_0$$
$$A = \sqrt[3]{12}\wp(\tau / \sqrt[3]{12}; g_2, g_3), \tag{2}$$
$$b_{11} = -b_{22} = \frac{1}{3}A - \frac{\kappa}{6}, b_{33} = b_{44} = \frac{1}{6}A - \frac{\kappa}{3}, b_{12} = b_{21} = 0.$$

It is indicated: g_2, g_3 are invariants of the Weierstrass function, and $g_2 = \kappa^2\sqrt[3]{12}$; τ_0 is a free parameter related to the choice of origin, $b_{ij} + b_{ji} - 2(\eta^{ij}b_{ij})\eta_{ij} = T_{ij}$ is the energy-momentum tensor of matter. Note that in this notation, the Einstein equations have the form

$$b_{ij} + b_{ji} + b\eta_{ij} = R_{ij} \tag{3}$$

$b = \eta^{ij}b_{ij}$; R_{ij} - Ricci tensor.

Suppose $g_2 = \sqrt[3]{12}$, $g_3 = 1$, then the half-periods of the Weierstrass function defined as $\omega_1 = 1.33003$, $\omega_2 = 0.66501 + 1.61260i$. Calculation of half-and the construction of appropriate 3D image of the Weierstrass function and its first derivative module carried out using the Wolfram Mathematica 9.0 [9].

In the metric (2) can be defined lattice defect such as a bubble. In the bubble we put $A^2 = \kappa^2$, while in the outer region the decision given in the form (2), therefore we have

$$A^2 = \kappa^2, e^{\nu} = 0, \quad |\tau| < \tau_0$$
$$A = \sqrt[3]{12}\wp(\tau / \sqrt[3]{12}, g_1, g_2), e^{\nu} = A_{\tau}, |\tau| > \tau_0 \quad (4)$$

On the borders of the bubble are continuous function A and its first derivative,

$$\kappa = \sqrt[3]{12}\wp(\tau_0 / \sqrt[3]{12}, g_1, g_2), A_{\tau} = 0, |\tau| = \tau_0 \quad (5)$$

In the particular case of a lattice with the invariants given in the form $g_2 = \sqrt[3]{12}, g_3 = 1$, we find the first zero and the corresponding value of the metric as follows: $\tau_0 = 3.0449983, \kappa = 2.1038034$. Note that the metric in the inner region of the bubble is a three-dimensional since it does not contain the radial coordinate. Indeed, using equation (1) and (4), we find

$$\Psi = -dt^2 + d\theta^2 + \cos^2(\sqrt{\kappa}\theta + \theta_0)d\varphi^2 \quad (6)$$

Similarly, the solution is constructed for the other roots of the second equation (5). All of these solutions are differ by the size of the bubble only, whereas the value κ does not change.

Any bubble can be turned inside out, just reversed inequality (4). In this case, we can extend the definition of the metric in the outer region bubble, using the solution of the first equation (2), so that the external space metric coincides with the metric of the universe [7]. Finally, the third type of particles can be formed as a combination of the first two and the result is a bubble which looks like a restricted shell of finite thickness - Fig. 1.

Note that the particles of the type shown in Figure 1, can be expanded at any rate, since this rate depends on the speed of the outer boundary, which can choose, including equal to the rate of expansion of the surrounding space. Hence, we find that there may be a spherical particle, which expand in sync with the space of the universe. Therefore, they appear to the outside observer static entities having spherical symmetry, such as protons.

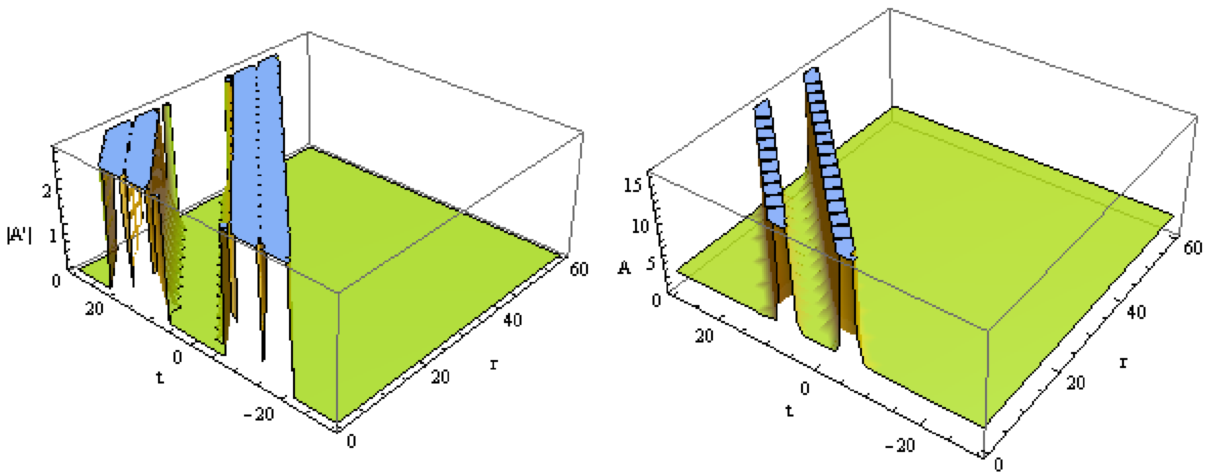

Figure 1: Bubble walls limited at $g_2 = \sqrt[3]{12}, g_3 = 1$.

Dynamics of quarks

Transform the metric (6) to standard form. To do this, multiply both sides of (6) on a constant $-\kappa$ and introduce new variables that differ from the old variables by a constant factor $\sqrt{\kappa}$, as a result we find

$$\Psi \to \Psi_1 = dt^2 - d\theta^2 - \sin^2\theta d\varphi^2 \tag{7}$$

The dynamics of the quarks in the inner region of the bubble with a metric of the form (7) we consider the system of Dirac equations in an external Yang-Mills field [10-11]. Regarding the Yang-Mills theory, we assume that the field in the inner region of the bubble is reduced to a set of constants. This model uses only two constants and the field itself is described by the vector potential

$$A^{\mu}_{YM} = (\Phi_0, 0, 0, A_0).$$

Note that according to (2) in the metric (7) energy-momentum tensor is constant. In addition, we consider the electromagnetic field, which generate quarks. Using the results of the paper [12], we transform the Dirac equation to the curvilinear coordinates (7). So we have system of equations

$$i\gamma^{\mu}(\nabla_{\mu} + iq_{ab}A^{b}_{\mu})\psi_a = m_{ab}\psi_a \tag{8}$$

Here indicated $\gamma^{\mu}, q_{ab}, A_{\mu}^{b}, \psi_{a}, m_{ab}$ - Dirac matrices, the interaction parameters, the vector potential, the wave function, and mass of quark *"a"* which is part of the particle *"b"*, respectively. Dirac matrices in the metric (7) have the form

$$\gamma^{0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^{\varphi} = \begin{pmatrix} 0 & 0 & 0 & -ie^{-i\varphi} \\ 0 & 0 & ie^{i\varphi} & 0 \\ 0 & ie^{-i\varphi} & 0 & 0 \\ -ie^{i\varphi} & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^{\theta} = \begin{pmatrix} 0 & 0 & -\sin\theta & e^{-i\varphi}\cos\theta \\ 0 & 0 & e^{i\varphi}\cos\theta & \sin\theta \\ \sin\theta & -e^{-i\varphi}\cos\theta & 0 & 0 \\ -e^{i\varphi}\cos\theta & -\sin\theta & 0 & 0 \end{pmatrix}$$

In this notation, the Dirac operator in the metric (7) can be written as

$$\gamma^{\mu}\nabla_{\mu} = \gamma^{0}\partial_{t} + \gamma^{\theta}\partial_{\theta} + \frac{\gamma^{\varphi}}{\sin\theta}\partial_{\varphi}$$

Since quarks have an electric charge, they generate an electromagnetic field through which interact with each other. To describe this interaction using the equations of quantum electrodynamics in the form of

$$\alpha q_{ab}\overline{\psi}_{a}\gamma^{\mu}\psi_{a} = (\partial_{t}^{2} - \nabla^{2})A_{e}^{\mu} \tag{9}$$

Here $\alpha = e^{2}/\hbar c$ is the fine structure constant, $\overline{\psi}_{a} = \psi_{a}^{+}\gamma^{0}, \psi_{a}^{+}$ is conjugated (by Hermit) vector. Thus, we assume that the currents and charges of the quarks are added, creating a collective field that quarks interact in accordance with equations (8).

The system of equations (8) - (9) was used to simulate the dynamics of quarks in the case of baryons, having the composition (uud), (udd), (sdu) - proton, neutron and baryon, respectively. In the simplest case, which takes into account only one electromagnetic field, the model contains 15 non-linear partial differential equations. To lower the order of the system we represent the solution of equations (8) - (9) in the form

$$\psi_a = e^{-i\omega t + iL\varphi} \begin{pmatrix} f_1(\theta) \\ f_2(\theta) e^{i\varphi} \\ if_3(\theta) \\ if_4(\theta) e^{i\varphi} \end{pmatrix}_a, \quad \begin{aligned} A_e^\mu &= (\Phi_e(\theta), 0, 0, A_e(\theta)) \\ A_b^\mu &= A_{YM}^\mu + A_e^\mu \end{aligned} \tag{10}$$

Here L, ω - the projection of the angular momentum on the selected axis and energy respectively. The system of the Dirac equations in the case of representation of the solution in the form (10), reduced to a form

$$\begin{aligned}
f_1' &= (L + q_{ab} A_b \sin\theta)(f_1 \cot\theta + f_2) + f_2 + \\
&(m_{ab} + \omega - q_{ab}\Phi_b)(f_3 \sin\theta - f_4 \cos\theta) \\
f_2' &= (L + q_{ab} A_b \sin\theta)(f_1 - f_2 \cot\theta) - f_2 \cot\theta - \\
&(m_{ab} + \omega - q_{ab}\Phi_b)(f_3 \cos\theta + f_4 \sin\theta) \\
f_3' &= (m_{ab} - \omega + q_{ab}\Phi_b)(f_1 \sin\theta - f_2 \cos\theta) + \\
&(L + q_{ab} A_b \sin\theta)(f_3 \cot\theta + f_4) + f_4 \\
f_4' &= -(m_{ab} - \omega + q_{ab}\Phi_b)(f_1 \cos\theta + f_2 \sin\theta) + \\
&(L + q_{ab} A_b \sin\theta)(f_3 - f_4 \cot\theta) - f_4 \cot\theta
\end{aligned} \tag{11}$$

Here we assume that $A_b = A_e + A_{YM}, \Phi_b = \Phi_e + \Phi_{YM}$. Thus, the fractional electric charge of a quark (but not its value, expressed in the electron charge!) is a measure of the interaction of quarks with the static Yang-Mills field. This hypothesis is not essential, since the introduction of a separate charge for describing the interaction of quarks with the Yang-Mills field reduces to a renormalization of the fields themselves.

Note that the mass and charge are individual for each quark, and the energy of the system are chosen to get standing waves along the meridian coordinate. The very existence of this type of solution is not obvious, since, for example, a similar three-body problem in classical mechanics with a pair interaction between particles leads to very complex solutions (so-called deterministic chaos). Calculating the current in the left-hand side of equation (9) and the del-operator in the right-hand side, we find

$$\alpha q_{ab} \overline{\psi}_a \gamma^0 \psi_a = \alpha q_{ab} \left(\sum_{i=1}^{4} f_i^2 \right)_a = -\Phi''_e - \Phi'_e \cot\theta \,, \qquad (12)$$

$$\alpha q_{ab} \overline{\psi}_a \gamma^\varphi \psi_a = 2\alpha q_{ab} (f_1 f_4 - f_2 f_3)_a = -A''_e - A'_e \cot\theta + \frac{A_e}{\sin^2\theta} \,,$$

$$\overline{\psi}_a \gamma^\theta \psi_a = 0 \,.$$

Here, with the index *"a"* is summation on all the quarks in the system. Note that the real charge density and the azimuthal component of the current rise to electric and magnetic fields, while the meridian component of the current vanishes on the solutions (10). Thus, in the case of baryons, the problem is reduced to solving a system of 14 ordinary differential equations. The boundary conditions for (11) - (12) we state as follows:

$$\begin{aligned} &f_1(0) = f_{1a}, f_2(0) = f_3(0) = f_4(0) = 0 \,|\, a = u, d, s; \\ &\Phi'_e(0) = 0, \Phi_e(0) = 0, A'_b(0) = 0, A_e(0) = 0. \end{aligned} \qquad (13)$$

Note that the boundary conditions (13) allow us to identify regular solution and eliminate the logarithmic singularity at the poles.

As is known, the electromagnetic properties of elementary particles are characterized by electric charge and magnetic moment. Therefore, the parameters of the Yang-Mills field, appearing in (11), should be related to the magnitude of the charge and magnetic moment of the quarks, which for the system are defined as follows

$$Q_b = \int dV q_{ab} \overline{\psi}_a \gamma^0 \psi_a = 4\pi \int_0^{\pi/2} d\theta \, \sin\theta \, q_{ab} \left(\sum_{i=1}^{4} f_i^2 \right)_a \qquad (14)$$

$$\mu_b = \frac{1}{2} \int dV [\mathbf{r} \times \mathbf{j}]_z = 2\pi \mu_q \int_0^{\pi/2} d\theta \, \sin^2\theta \, q_{ab} \overline{\psi}_a \gamma^\varphi \psi_a =$$

$$4\pi \mu_q \int_0^{\pi/2} d\theta \, \sin^2\theta \sum_a q_{ab} (f_1 f_4 - f_2 f_3)_a$$

We take 1 MeV as unit of mass, and then the parameters of the Yang-Mills field, the vector potential and the energy of the system will be expressed in units of

MeV. The unit of the magnetic moment in this case is $\mu = e\hbar / MeV = 2m_e \mu_B \approx 1.0219978\, \mu_B$, where μ_B is the Bohr magneton. Here is a numerical factor equal to twice electron mass, expressed in the unit of mass. Consequently, the unit of the magnetic moment in this system is the Bohr magneton, and not nuclear magneton, as proposed in the first papers on the theory of the magnetic moments of baryons composed of quarks [10, 13-14].

Note that the prediction of the anomalous magnetic moments of baryons was a great success of the theory of SU (6), which is an indirect confirmation of the hypothesis of the existence of quarks, as components of hadrons [10]. To calculate the magnetic moments the non-relativistic theory and the hypothesis of a large mass of free quarks $m_q \approx 4GeV$ been used. Later, however, it turned out that the mass of the quarks that make up the nucleons of a few MeV, which baffled all the original theory. At present, the calculations of the magnetic moments of baryons are made on the basis of highly complex numerical models LQCD [4-6].

Let determine the distribution of the current density and magnetic moment

$$j(\theta) = \sum_a 2\alpha\, q_{ab} \left(f_1 f_4 - f_2 f_3 \right)_a \tag{15}$$

$$\mu(\theta) = 4\pi \mu_q \int_0^\theta d\theta\, \sin^2\theta \sum_a q_{ab} \left(f_1 f_4 - f_2 f_3 \right)_a$$

The wave functions of the quarks can be normalized in the standard way as:

$$1 = \int dV \overline{\psi}_a \gamma^0 \psi_a = 4\pi \int_0^{\pi/2} d\theta\, \sin\theta \left(\sum_{i=1}^4 f_i^2 \right)_{a=u,d,s} \tag{16}$$

With this method of normalization quarks are considered as real particles that are present in the other particle in a given proportion. Since free quarks are not observed, the question of the validity of the normalization (16) remains open.

We can assume that a complete model of baryons contain, along with the spin, electric charge and magnetic moment, mass and lifetime of the particles, and a description of the excited states, which in this model corresponds to the spectrum of the energy of the quarks.

The magnetic moments of baryons

In model (7) - (14) the calculation of the magnetic moments is reduced, as shown above, the definition of the two parameters characterizing the Yang-Mills field in the inner region of the bubble, with a given total energy of the system. This task was computed as the numerical model, implemented in the Wolfram Mathematica 9.0 [9]. It was found that the scale of parameters of the Yang-Mills field is less than 1 MeV. Hence, one of the model parameters, for example, Φ_{YM} can be excluded from consideration, since this parameter involved in a linear combination with the quark masses, which are defined around with such precision. On the other hand, the energy of a single quark can be set equal to, for example, the mass of the neutral pi meson, i.e. put

$$\omega = m_{\pi} \approx 134.9766 MeV . \qquad (17)$$

In this case, there is a clear relationship between the magnitude of the potential of the Yang-Mills field in the inner region of the bubble, and the magnetic moment of the particle.

General properties of the studied baryons and quarks are presented in Tables 1-2.

Table 1: Properties of baryons

Symbol	Spin	Charge	Mass	BaryonNumber	GFactor	Hypercharge	Isospin	QuarkContent
p	$\frac{1}{2}$	1	938.27203	1	5.585694713	1	$\frac{1}{2}$	{{DownQuark, UpQuark, UpQuark}}
$\bar{p}$	$\frac{1}{2}$	−1	938.27203	−1	5.585694713	−1	$\frac{1}{2}$	{{DownQuarkBar, UpQuarkBar, UpQuarkBar}}
n	$\frac{1}{2}$	0	939.56536	1	−3.82608545	1	$\frac{1}{2}$	{{DownQuark, DownQuark, UpQuark}}
$\bar{n}$	$\frac{1}{2}$	0	939.56536	−1	−3.82608545	−1	$\frac{1}{2}$	{{DownQuarkBar, DownQuarkBar, UpQuarkBar}}
Λ	$\frac{1}{2}$	0	1115.683	1	−1.226	0	0	{{StrangeQuark, DownQuark, UpQuark}}
$\bar{\Lambda}$	$\frac{1}{2}$	0	1115.683	−1	−1.226	0	0	{{StrangeQuarkBar, DownQuarkBar, UpQuarkBar}}

In Figure 2 shows the results of modeling the structure of the proton - the current, distributed parameters of the magnetic moment and the wave functions. For the parameters of the quarks, shown in Table 2, the following values

$$\omega = 134.9766 MeV, \; A_{YM} = -0.617 MeV, \; \Phi_{YM} = 0,$$
$$L_d = -\frac{1}{2}, \; f_{1d}(0) = 22.9395, \; L_u = \frac{1}{2}, \; f_{1u}(0) = 0.3077 \tag{18}$$

Thus, the proton potential of the Yang-Mills field in a bubble is indeed relatively small. The total energy of the quarks in this state is $3\omega = 3m_\pi \approx 404.93 MeV$, the total momentum of the system is equal to the proton spin and magnetic moment equal to the magnetic moment of the proton with the experimental accuracy. Under the conditions (18) is also valid the normalization condition (16), so that the total charge of the quark equal to the proton charge.

Table 2: Properties of quarks

Symbol	Spin	Charge	Mass	BaryonNumber	Bottomness	Charm	Hypercharge	Isospin	Strangeness	Topness
u	$\frac{1}{2}$	$\frac{2}{3}$	2.2	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{2}$	0	0
$\bar{u}$	$\frac{1}{2}$	$-\frac{2}{3}$	2.2	$-\frac{1}{3}$	0	0	$-\frac{1}{3}$	$\frac{1}{2}$	0	0
d	$\frac{1}{2}$	$-\frac{1}{3}$	5.0	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{2}$	0	0
$\bar{d}$	$\frac{1}{2}$	$\frac{1}{3}$	5.0	$-\frac{1}{3}$	0	0	$-\frac{1}{3}$	$\frac{1}{2}$	0	0
s	$\frac{1}{2}$	$-\frac{1}{3}$	95.	$\frac{1}{3}$	0	0	$-\frac{2}{3}$	0	−1	0
$\bar{s}$	$\frac{1}{2}$	$\frac{1}{3}$	95.	$-\frac{1}{3}$	0	0	$\frac{2}{3}$	0	1	0
c	$\frac{1}{2}$	$\frac{2}{3}$	1250.	$\frac{1}{3}$	0	1	$\frac{4}{3}$	0	0	0
$\bar{c}$	$\frac{1}{2}$	$-\frac{2}{3}$	1250.	$-\frac{1}{3}$	0	−1	$-\frac{4}{3}$	0	0	0
b	$\frac{1}{2}$	$-\frac{1}{3}$	4200.	$\frac{1}{3}$	−1	0	$\frac{1}{3}$	0	0	0
$\bar{b}$	$\frac{1}{2}$	$\frac{1}{3}$	4200.	$-\frac{1}{3}$	1	0	$-\frac{1}{3}$	0	0	0
t	$\frac{1}{2}$	$\frac{2}{3}$	174 200.	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	0	1
$\bar{t}$	$\frac{1}{2}$	$-\frac{2}{3}$	174 200.	$-\frac{1}{3}$	0	0	$-\frac{1}{3}$	0	0	−1

Note that in this model the magnetic moment is calculated by the standard formulas of electrodynamics (14), and not as a quantum value. Therefore the magnetic moment of the proton is determined not magneton of quarks, as suggested in the models [10, 13-14], but it depending on the current, which add up to the observed value of the anomalous magnetic moment.

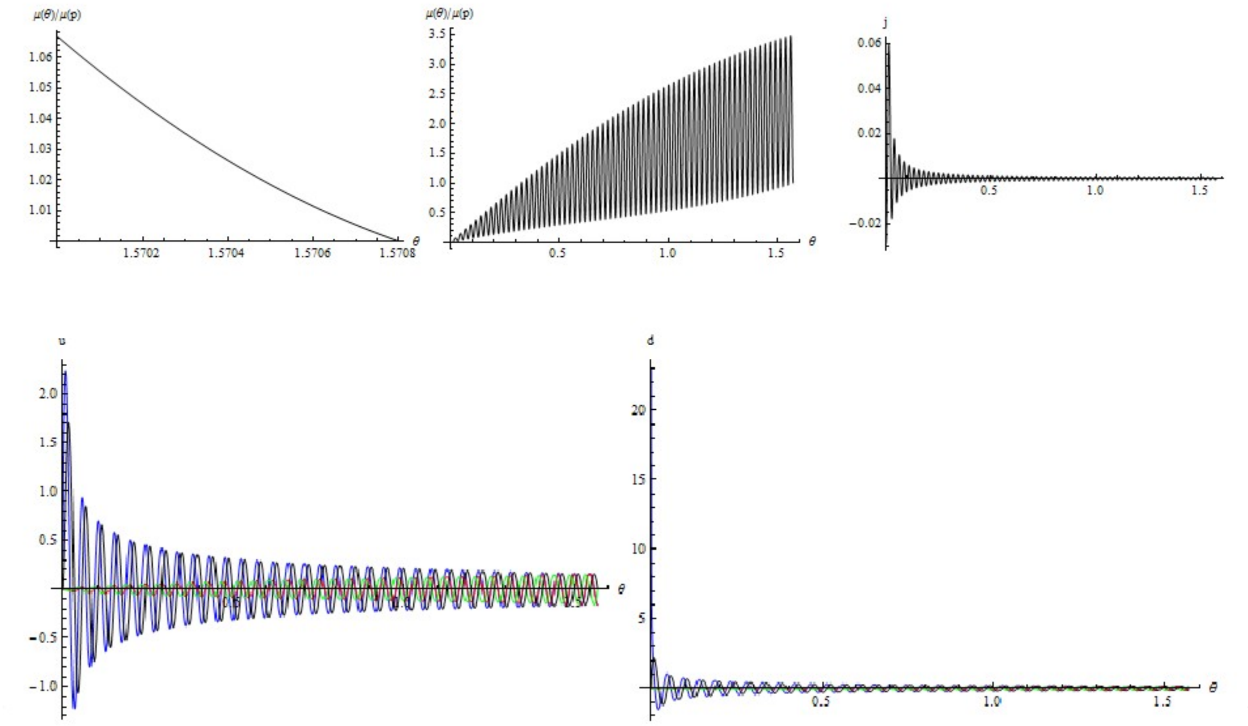

Figure 2: The magnetic moment of the quark (uud) normalized to the magnetic moment of the proton, the electromagnetic current in the system and the wave functions of the quarks.

In Figure 3 the results of modeling the structure of the neutron are shown, which can be compared with similar data for the proton - Figure 2. Note that the data for the distribution of the magnetic moment of the neutron and proton are normalized to their experimental values given in Table 1 (note, there are g-factors only in column "GFactor" which obviously connected to the nuclear magneton). For the neutron, the following values of the model parameters found:

$$\omega = 134.9766 MeV,\ A_{YM} = -.0666 MeV,\ \Phi_{YM} = 0,$$
$$L_d = \frac{1}{2},\ f_{1d}(0) = 0.3092,\ L_u = -\frac{1}{2},\ f_{1u}(0) = 22.6882. \qquad (19)$$

Note that the potential of the Yang-Mills field in the case of the neutron is negative, and by an order of magnitude less than that of the proton potential. The total energy of the quarks in the neutron at the above values of the parameters is $3\omega = 3m_\pi \approx 404.93 MeV$. If conditions (19) are true, the magnetic moment of the quark system equal to magnetic moment of the neutron with the experimental accuracy. Fulfilled the conditions for the normalization (16), so the charge of the system is zero, and the spin system is ½.

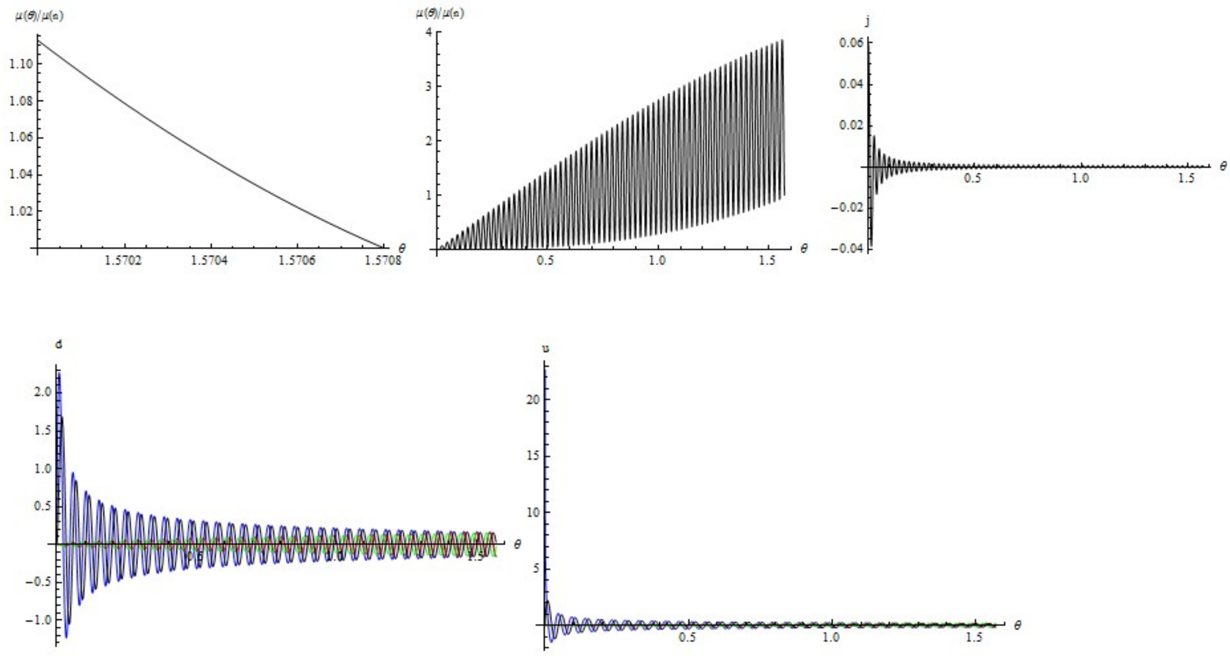

Figure 3: The magnetic moment of quarks (udd) normalized to the magnetic moment of the neutron, the electromagnetic current in the system and the wave functions of the quarks.

In Fig. 4 presents the modeling structure of the lambda baryon. In this case, the following values of the model parameters are calculated

$$\omega = 134.9766 MeV,\ A_{YM} = -0.0425 MeV,\ \Phi_{YM} = 0,$$
$$L_d = -\frac{1}{2},\ f_{1d}(0) = 22.938,\ L_s = L_u = \frac{1}{2}, \quad (20)$$
$$f_{1s}(0) = 0.28151,\ f_{1u}(0) = 0.30632$$

For the parameters (20) the magnetic moment of the quark system (sdu) is equal to the magnetic moment of the lambda baryon with experimental error. The potential of the Yang-Mills equations for a system of quarks (sdu) is negative and smaller in magnitude than that of the neutron and the order of magnitude smaller than that of the proton. For this system, the normalization conditions (16) are true, the charge of the system is zero, and the spin is ½.

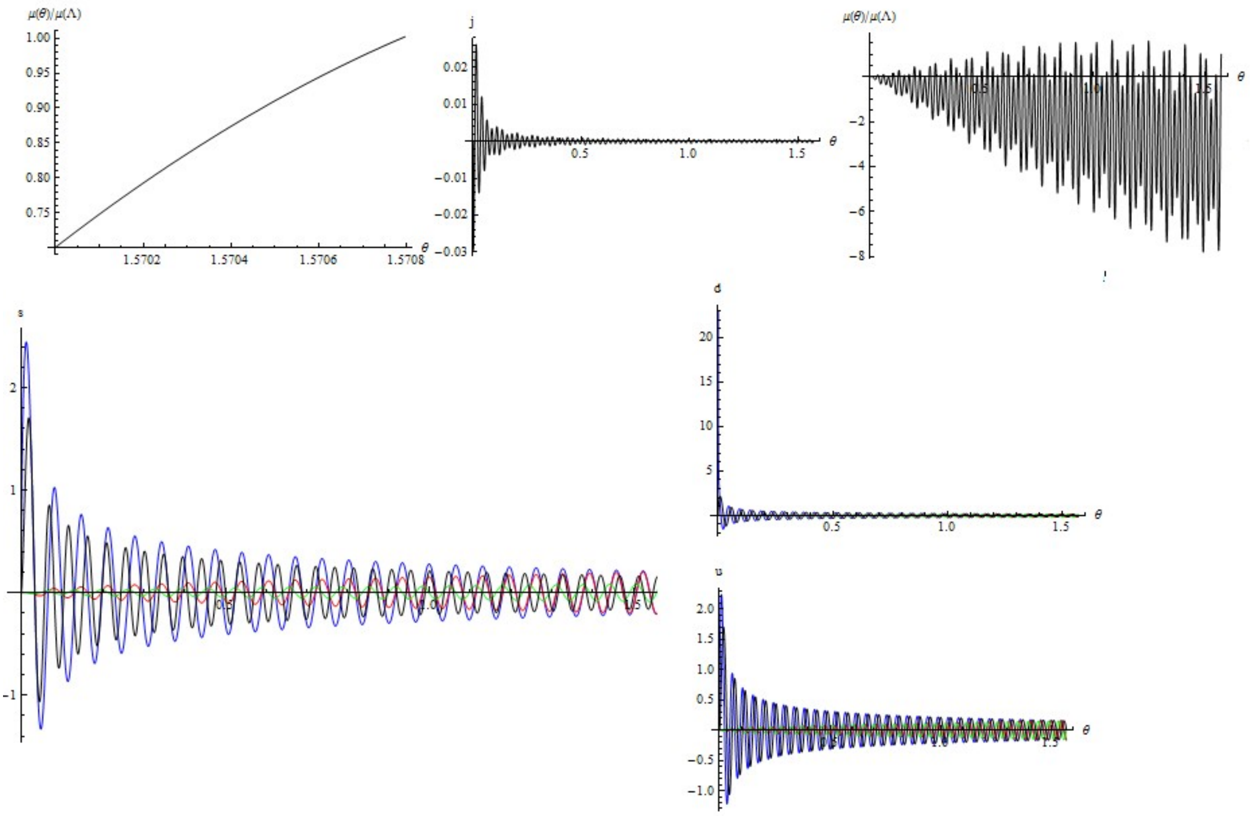

Figure 4: The magnetic moment of quarks (sdu) normalized to the magnetic moment of the lambda baryon, the electromagnetic current in the system and the wave functions of the quarks.

Thus, we have shown that the quarks in the metric system of hadrons can be described on the basis of the Dirac equation and the equations of quantum electrodynamics. The closed model (8) - (14) formulated and the magnetic moments of hadrons (uud), (udd) and (sdu) at given energy and given electric charge are calculated. The investigated region corresponds to the resonance energy of the quarks system, in which, apparently, pi mesons can be generated.

Finally, we note that the above model of the dynamics of quarks in hadrons metric (7) has an interesting property: the quarks do not leave the inner region of the bubble, while the spherical symmetry is not broken. Indeed, the motion of the quarks is fully implemented in three-dimensional space of the metric (7). It is necessary to transfer the radial momentum to force the quarks move outside the bubble, but it is impossible in the metric (7). The change in the metric is only possible with a significant perturbation of the main Yang-Mills field.

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