Chaos and Correlation
International Journal

Chaos and Correlation
International Journal, February 1, 2012

Структура атомного ядра в теории Калуцы-Клейна

The structure of atomic nuclei in Kaluza-Klein theory

Alexander P. Trunev (Toronto, Canada)

Alexander P. Trunev

На основе теории Калуцы-Клейна изучены особые состояния, возникающие при взаимодействии протонов со скалярным безмассовым полем. Показано, что некоторые состояния имеют параметры атомных ядер. Вычислена зависимость энергии связи от числа нуклонов для всей совокупности известных нуклидов.
.

The special states, arising from the interaction of protons with a scalar massless field studied on the basis of Kaluza-Klein theory. It is shown that some states have the parameters of atomic nuclei. We calculate the binding energy dependence on the number of nucleons for the entire set of known nuclides.

Ключевые слова: нейтрон, протон, электрон, ядро.

Keywords: Electron, Proton, Neutron, Nuclei.

Introduction

Nuclear shell model [1-3] is widely used to describe processes in atomic nuclei. As is known, the nuclear shell model is based on an analogy with the electron shells of atoms. The question arises whether it is possible to construct a model of the nucleus to the electronic and atomic shells were described by one equation? In this model, for example, may find an explanation for the beta-decay and K capture, the transitions from one energy level to another.

The above issue is closely related to the problem of the origin of elementary particles. It was established earlier [4] that the effect of electromagnetic field on the metric in the five-dimensional space in the vicinity of a charged center of gravity leads to bound states of the scalar field describing the mass spectrum of elementary particles. In the papers [5-6] we solve the problem of the structure of the hydrogen atom and a neutron. In particular, it has been shown [5] that the interaction of protons with a scalar massless field can be formed of a particle with a mass close to the mass of the neutron.

We suppose [4-6] that massless scalars field, for instance, the scalar potential of electromagnetic field [7-9], acts as the universal substance, which forms the elementary particles and atoms. In this paper we studied the problem of constructing a universal model of the atom and atomic nucleus based on the theory of fundamental interactions [4]. We study the interaction of the cluster with the total number of protons A=N+Z with a massless scalar field. Set the bound states with the parameters of atomic nuclei with charge Z, containing A = Z + N nucleons. We calculate the binding energy dependence on the number of nucleons for the entire set of known nuclides.

The Master Equation

To describe the motion of matter in the light of its wave properties, we assume that the standard Hamilton-Jacobi equation in the relativistic mechanics and the equation of the Klein-Gordon equation in quantum mechanics arise as a consequence of the wave equation in five-dimensional space [4-7]. This equation can generally be written as:

$$\frac{1}{\sqrt{-G}}\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-G}G^{\mu\nu}\frac{\partial}{\partial x^{\nu}}\Psi\right)=0 \tag{1}$$

Here Ψ - the wave function describing, according to (1), the massless scalar field in five-dimensional space; G^{ik} - the contravariant metric tensor,

$$G^{ik}=N^{-1}\begin{pmatrix}\lambda_1 & 0 & 0 & 0 & -g^1\\ 0 & \lambda_2 & 0 & 0 & -g^2\\ 0 & 0 & \lambda_2 & 0 & -g^3\\ 0 & 0 & 0 & \lambda_2 & -g^4\\ -g^1 & -g^2 & -g^3 & -g^4 & \lambda\end{pmatrix} \tag{2}$$

$$\lambda_1=(1-\varepsilon^2/kr)^{-1};\quad \lambda_2=-(1+\varepsilon^2/kr)^{-1}$$
$$g^1=\lambda_1 g_1,\ g^2=\lambda_2 g_2,\ g^3=\lambda_2 g_3,\ g^4=\lambda_2 g_4\ ;$$

$$\lambda=1+\lambda_1 g_1^2+\lambda_2(g_2^2+g_3^2+g_4^2);\quad G=N^5/(ab^3);\quad N=(kr)^2.$$

Vector potential of the source associated with the center of gravity, has the form

$$g_1=\varepsilon/kr,\quad \mathbf{g}=g_1\mathbf{u} \tag{3}$$

Here $\mathbf{u}$ - a vector in the three dimensional space, which we define below. In particular, the scalar and vector potential of electromagnetic field of a single charge of mass *m* can be written as

$$\varphi_e=\frac{q}{r}=\frac{mc^2}{e}\frac{\varepsilon}{kr},\quad \mathbf{A}=\varphi_e\mathbf{u} \tag{4}$$

In this case we have: $k=2\gamma m^3c^2/e^4,\quad \varepsilon^2/k=2\gamma m/c^2$, γ is the gravitational constant. The numerical value of the parameter k, has dimension of inverse length, is in the case of an electron around $1.7*10^{-28}$ m^{-1}, and in the case of the proton about $1.05*10^{-18}$ m^{-1}. Note that the corresponding scale in the case of an electron exceeds the size of the observable universe, while for protons this scale is about 100 light-years – see Table 1.

Table 1: The parameters of the metric tensor

	k, 1/m	ε	r_{max}, m	r_{min}, m
e-	1.703163E-28	4.799488E-43	5.87E+27	2.81799E-15
p+	1.054395E-18	1.618178E-36	9.48E+17	1.5347E-18

Equation (1) is interesting because of it, by a simple generalization; we can derive all the basic models of quantum mechanics, including the Dirac equation, as in the nonrelativistic case, this equation reduces to the Schrodinger equation. From this we can also derive the eikonal equation, which is 4-dimensional space is reduced to the Hamilton-Jacobi equation, which describes the motion of relativistic charged particles in electromagnetic and gravitational field [4].

We further note that in the investigated metrics, depending only on the radial coordinate, the following relation

$$F^{\mu} = N\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-G}G^{\mu\nu}\right) = N\frac{\partial r}{\partial x^{\mu}}\frac{d}{dr}\left(\sqrt{-G}G^{\mu\nu}\right) \tag{5}$$

Taking into account expressions (2), (5), we write the wave equation (1) as

$$\frac{\lambda_1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - |\lambda_2|\nabla^2\Psi + \lambda\frac{\partial^2\Psi}{\partial\rho^2} - 2g^i\frac{\partial^2\Psi}{\partial x^i\partial\rho} + F^{\mu}\frac{\partial\Psi}{\partial x^{\mu}} = 0 \tag{6}$$

Note that the last term in equation (6) is of order $N^2k = k^5r^4 \ll 1$. Consequently, this term can be dropped in the problems, the characteristic scale which is considerably less than the maximum scale in Table 1. Equation (6) is remarkable in that it does not contain any parameters that characterize the scalar field. The field acquires a mass and charge (not just electrical, but also strong and the weak [4]) in the process of interaction with the central body, which is due only to the metric and five-dimensional space.

The spectrum of atomic particles with axial symmetry

Consider the problem of the motion of matter around the charged center of gravity, which has an electrical charge and strong, for example, around the proton. In the process of solving this problem is necessary to define the inertial mass of matter and energy ties. Since equation (6) is linear and homogeneous, this problem can be solved in general case.

We introduce a polar coordinate system (r,ϕ,z) with the z axis is directed along the vector potential (4), we put in equation (6)

$$\Psi = \psi(r)\exp(il\phi + ik_z z - i\omega t - ik_\rho\rho) \tag{7}$$

Separating the variables, we find that the radial distribution of matter is described by the following equation (here we dropped, because of its smallness, the last term in equation (6)):

$$-\frac{\lambda_1\omega^2}{c^2}\psi - |\lambda_2|\left(\psi_{rr} + \frac{1}{r}\psi_r - \frac{l^2}{r^2}\psi - k_z^2\psi\right) - \lambda k_\rho^2\psi + 2g^1c^{-1}\omega k_\rho\psi - 2g^z k_z k_\rho\psi = 0 \qquad (8)$$

We assume that the characteristic scale of the spatial distribution of matter far beyond the gravitational radius, $r >> \varepsilon^2/k = 2\gamma M/c^2$. Then, in first approximation we can assume that $\lambda_1 \approx -\lambda_2 \approx 1;\ \ \lambda = 1 + g_1^2 - \mathbf{g}^2 \approx 1$. We also use the definition of the vector and scalar potential (3), as a result we obtain

$$\psi_{rr} + \frac{1}{r}\psi_r - \frac{l^2}{r^2}\psi - k_z^2\psi + \left(K^2 + \frac{\kappa_g}{r}\right)\psi = 0$$
$$K^2 = k_\rho^2 + \omega^2/c^2, \quad \kappa_g = -2\varepsilon k_\rho (k_z u_z + \omega/c)/k > 0 \qquad (9)$$

Note that equation (9) coincides in form with what was obtained in [8-9] in the case of axially symmetric solutions of the Schrödinger equation describing the special states of the hydrogen atom. We seek the solution of equation (9) as

$$\psi = \psi_0 \frac{\exp(-r/r_0)}{r^a} \qquad (10)$$

Substituting (19) in equation (18), we find

$$\frac{a^2 - l^2}{r^2} + \frac{2a - 1 + r_0\kappa_g}{rr_0} + \frac{1}{r_0^2} - k_z^2 + k_\rho^2 + \frac{\omega^2}{c^2} = 0 \qquad (11)$$

Equating coefficients of like powers of r, we find the equation for determining the unknown parameters:

$$a = \pm l, \quad r_0 = \frac{1 - 2a}{\kappa_g}, \quad \frac{1}{r_0^2} - k_z^2 + k_\rho^2 + \frac{\omega^2}{c^2} = 0 \qquad (12)$$

The second equation (12) holds only for values of the exponent, for which the inequality $a < 1/2$ is true. Hence, we find an equation for determining the frequency

$$\frac{4\varepsilon^2 k_\rho^2}{k^2(2l+1)^2}\left(k_z u_z + \frac{\omega}{c}\right)^2 - k_z^2 + k_\rho^2 + \frac{\omega^2}{c^2} = 0 \qquad (13)$$

It should be noted that the original metric in the five-dimensional space defined by metric tensor, which depends only on the parameters of the central body, ie the charge and mass of the proton, so the first term of the left side of equation (13) should be placed $\varepsilon/k = e^2/m_p c^2$.

The structure of the neutron

The experimentally determined main properties of the neutron are shown in Table 2. The average lifetime of a free neutron is about to 885.7 c. A neutron decays into a proton, electron and anti-neutrino in the scheme (beta decay):

$$n^0 \to p^+ + e^- + \bar{\nu}_e \tag{14}$$

Table 2: The fundamental properties of elementary particles involved in the reaction (35) [10].

Elementary particle	Neutron n^0	Proton p^+	Electron e^-	$\bar{\nu}_e$
Mass, MeV/c2	939.565560(81)	938.272013(23)	0.510998910(13)	< 2.2 eV
Charge, Coulomb	0	$1.602176487(40)\times10^{-19}$	$-1.602176487(40)\times10^{-19}$	0
The magnetic moment, nuclear magneton (N) or Bohr magneton (B)	−1.9130427(5) μN	2.792847351(28) μN	−1.00115965218111 μB	10^{-19} μ_B
Electric dipole moment	$<2.9\times10^{-26}$ e.cm	$<5.4\times10^{-24}$ e.cm	?	?

It was found that the proton in the nucleus may be transferred in accordance with the scheme of the neutron inverse beta-decay

$$p^+ \to n^0 + e^+ + \nu_e \tag{15}$$

Another possible channel is the electron capture:

$$p^+ + e^- \to n^0 + \nu_e \tag{16}$$

The first theory of beta decay proposed in 1933 by Enrico Fermi. Later it was suggested several theories, including the theory of Feynman and Gell-Mann [11]. At the present time, according to the present standard model, the reaction (14) comes with the participation of the intermediate vector gauge boson W^- [12]. In this model, protons and neutrons are composite particles containing three quarks on. However, protons are split into their component parts have failed, although it is believed that hadronic jets observed in experiments on collisions of protons at high energy, are the quark-gluon plasma [13].

It was found that the distribution of electric charge in the neutron consists of a negatively charged outer coat, inner layer of positively charged and negatively charged nucleus [14]. From the decay scheme (14) and classical representations of the interaction of charged particles, one would assume that the proton forms together with the electron kind of hydrogen atom, which explains the observed electromagnetic structure of the neutron [9]. But we know that the state describing the hydrogen atom with a large binding energy, consistent hydrino [15-17]. In these states, the mass of the hydrogen atom is different from that of the proton by a small amount $\alpha m_e c^2$ that is not consistent with the large mass of neutron, exceeding the total mass of a proton and an electron by an amount $(m_n - m_p - m_e)/m_e = 1{,}531015$.

Let us consider the state of five-dimension hydrogen atom, which correspond to the parameters of the neutron and proton in Table 2. In this case, there are no similar solutions, which are described a neutron on the basis of relativistic Dirac or Klein-Gordon equation. The wave vector of the fifth dimension can be determined from the third equation (12), as a result we find

$$S = \frac{P^2 - E^2}{1 + b(Pu + E)^2}, \quad k_\rho = \pm \frac{m_e c}{\hbar}\sqrt{S}$$
$$b = \frac{4\varepsilon^2}{\hbar^2 k^2}\frac{m_e^2 c^2}{(1-2a)^2}, \quad P = \frac{\hbar k_z}{m_e c}, \quad E = \frac{\hbar\omega}{m_e c^2} \qquad (17)$$

Surface, which is given by the first equation (17), depends on the interaction parameter, which in turn depends on the type of interaction. In general, we can set $\varepsilon / k = e^2 / m_p c^2$, but the square of the charge can take three values of [4], which correspond to the electromagnetic, strong and weak interactions, respectively - see Table 3.

Table 3: The parameter b for the three types of interaction with a = 0

Interaction type	Charge	Parameter of Interaction, b
Electromagnetic	$e^2 = \alpha \hbar c$	6.3179E-11
Strong	$e_s^2 = e^2 (m_p / m_e)^{3/2}$	4.97091E-06
Weak	$e_w^2 = e^2 (m_e / m_p)^{3/2}$	8.03E-016

As follows from Table 3, the effect of the interaction parameter on the dispersion relation, even in the case of strong interaction manifests itself at energies of about 300 electron masses. There is however a special case where $a \to 1/2$. Then, it follows from (17) the interaction parameter can take any value. In this particular case, all interactions are compared with each other in the sense that there always exists a value of the exponent, for any type of interaction we have the product parameters $Sb \approx 1$.

The surface $S = S(P, E)$ for parameter values $b = 0{,}078051; u = u_z = -1$ is shown in Figure 1. Each section of the surface for positive values of S determines the line of dispersion relation $E = E(P, S)$.

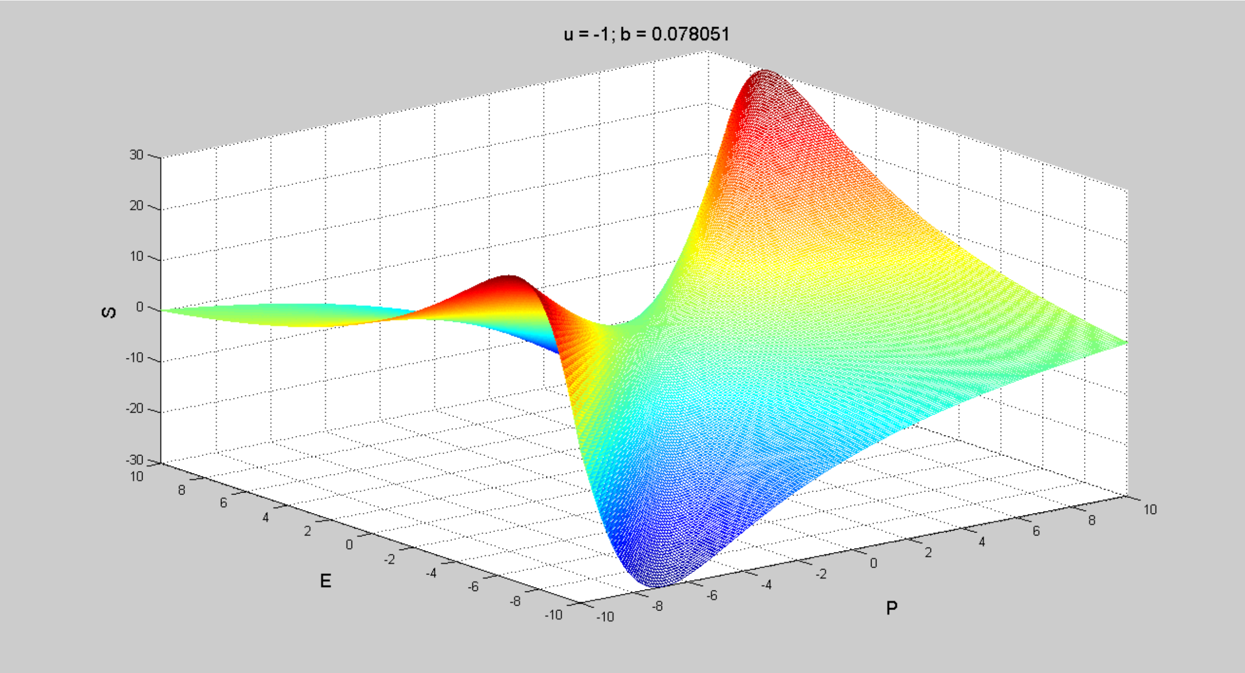

Figure 1: Surface, which characterizes the special states of a hydrogen atom at $b = 0{,}078051; u = u_z = -1$.

Note that a special case $a = 1/2$ was considered for the Schrodinger equation in [7] and for equation (6) in [6]. The common property of these states lies in the fact that the electron approaches the nucleus for a short distance of the order of the classical electron radius. For example, in the model [9] we have

$$r_0 / r_e = 0.4777778, \quad \mathrm{r_e} = e^2 / m_e c^2, \quad L = 1.376791 \alpha \hbar \tag{18}$$

Consider the dispersion relation that characterizes this condition. Solving the first equation (17) with respect to energy, we find the dispersion relation - Fig. 2, which allows determining the minimum energy and momentum of a scalar field in a particular state, using the conditions for the ascending part of the spectrum:

$$E_m = (m_n - m_p) / m_e \approx 2.531015; \lim_{\mathrm{P} \to \infty} \mathrm{E/P} = 1$$

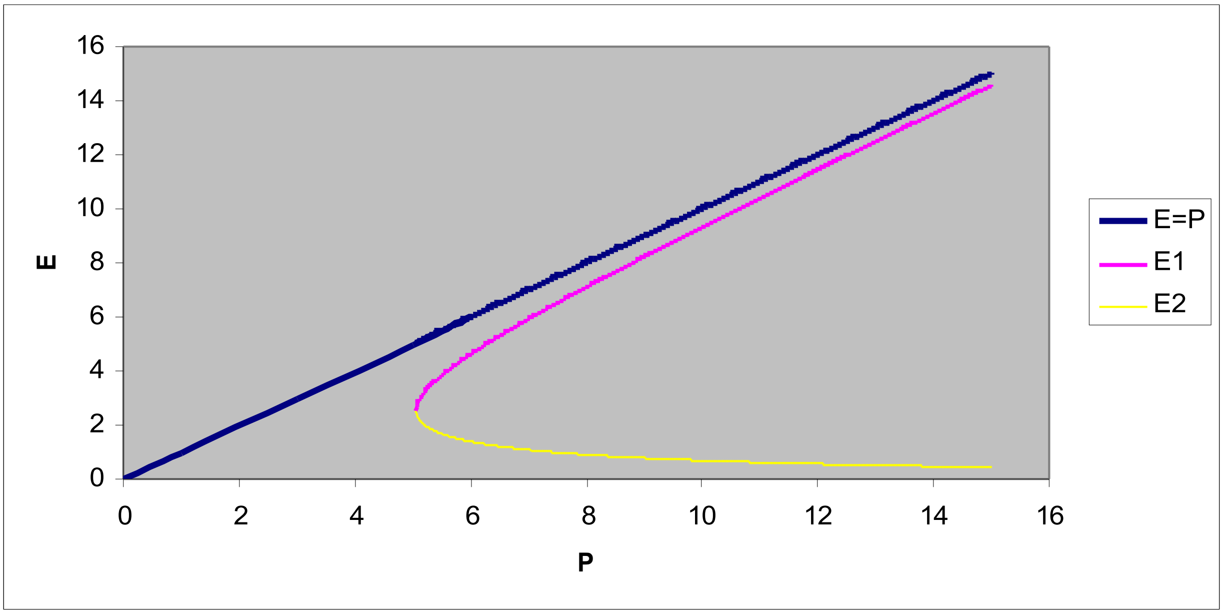

Figure 2: Energy-momentum dispersion relation which characterizes the special states of a hydrogen atom at $b = 0{,}078051; u = u_z = -1$.

These conditions allow us to determine the numerical values of other parameters. Indeed, set $Sb = -u = 1$ in the first equation (17), taking into account these relations we rewrite the above equation as:

$$S + P^2 - 2PE + E^2 = P^2 - E^2$$

Solving this equation, we find

$$E = \frac{P}{2} \pm \sqrt{\frac{P^2}{4} - \frac{S}{2}}$$

Hence, if we put $P^2 >> S$ for the ascending branch of solutions we have $E = P - S/2P + \ldots$. From the last expression it follows $E/P = 1$ in the limit of $P \to \infty$. The minimum value of the momentum is determined from the condition $P_m^2 = 2S$, therefore $P_m = 2E_m$. Hence we find the parameters of state

$$E_m = 2.531015; \mathrm{P_m} = 5.06203; S = 12.81208; \mathrm{b} = 0.078051 \qquad (19)$$

Note that the dispersion curves in Fig. 2, describing the special states of a hydrogen atom, contain ascending and descending branch, as well as the maximum limit of the spectrum. For the excitation of these states must provide a minimum momentum. The size of the hydrogen atom in this state is determined by the Compton wavelength of an electron:

$$r_0 / \lambda_e = 1/E \approx 0{,}395098, \quad \lambda_e = \hbar / m_e c \qquad (20)$$

As we know, the state of the hydrogen atom with a characteristic scale (20), associated with gydrino [5-8,15-17]. First, these states were obtained by Sommerfeld in 1923 as a solution of the Klein-Gordon equation for the relativistic hydrogen atom. Note that the Sommerfeld solution can be obtained from equation (6) in the absence of magnetic interaction at $\lambda = 1 + g_1^2; \mathbf{g} = 0$. At present there is not only a theory, but a lot of experiments confirming the hypothesis of the existence of special states of the hydrogen atom - gidrino [17]. Solution obtained above is a generalization of known results [15-16] to the case of magnetic interaction due to a special metric in the five-dimensional space [4, 18].

You may notice that the dispersion relation $S = S(P,E)$ obtained by section of the surface shown in Fig. 1, is invariant with respect to the choice of scale. Therefore, choosing a scale of the classical electron radius, we obtain

$$r_0 / r_e = 1/E \approx 0{,}395098, \quad r_e = e^2 / m_e c^2 \tag{21}$$

This result is consistent with the data (18), but the final choice of scale in the model depends on the determination of the neutron magnetic moment [15], which is beyond the scope of this paper.

Thus, we have shown that there are specific states of the hydrogen atom, which describe a particle with mass and size of the neutron. These states arise in the interaction of protons with a massless scalar field. The resulting interaction density distribution of the scalar field corresponds to the Yukawa potential

$$\psi^2 = \psi_0^2 \frac{\exp(-2r/r_0)}{r^{1-\delta}}, \delta = 1 - 2a << 1$$

Further study of this problem may be related with the influence of the scalar field on a standard metric in the Kaluza-Klein theory [18].

The structure of atomic nuclei

We can assume that if the sum of the protons interacting with a scalar field, they can form an atom, consisting of the electron shell and nucleus with electric charge eZ, number of nucleons $A = Z + N$ and the mass defect $\Delta M = m_p Z + m_n N - E_b / c^2$, where E_b - the energy of the nucleons in the nucleus.

In Fig. 3 shows the dependence of the mass excess, obtained by processing data [19] for all known 3179 nuclides (including neutron and proton). Note that mass excess $ME = M - A$ is the binding energy in units of energy of the nucleons in the nucleus of the carbon isotope ^{12}C, for which we have $ME = M - 12 = 0$ - blue surface in Fig. 3. The binding energy can be expressed in the form

$$E_b = Z(m_e + m_p) + Nm_n - (ME + A \cdot m_u),$$
$$m_u = m(^{12}C)/12 \approx 931.494028MeV \quad (22)$$

Here, in the second equation provides a definition of atomic mass unit, which is used for tabulation of data in nuclear physics and chemistry.

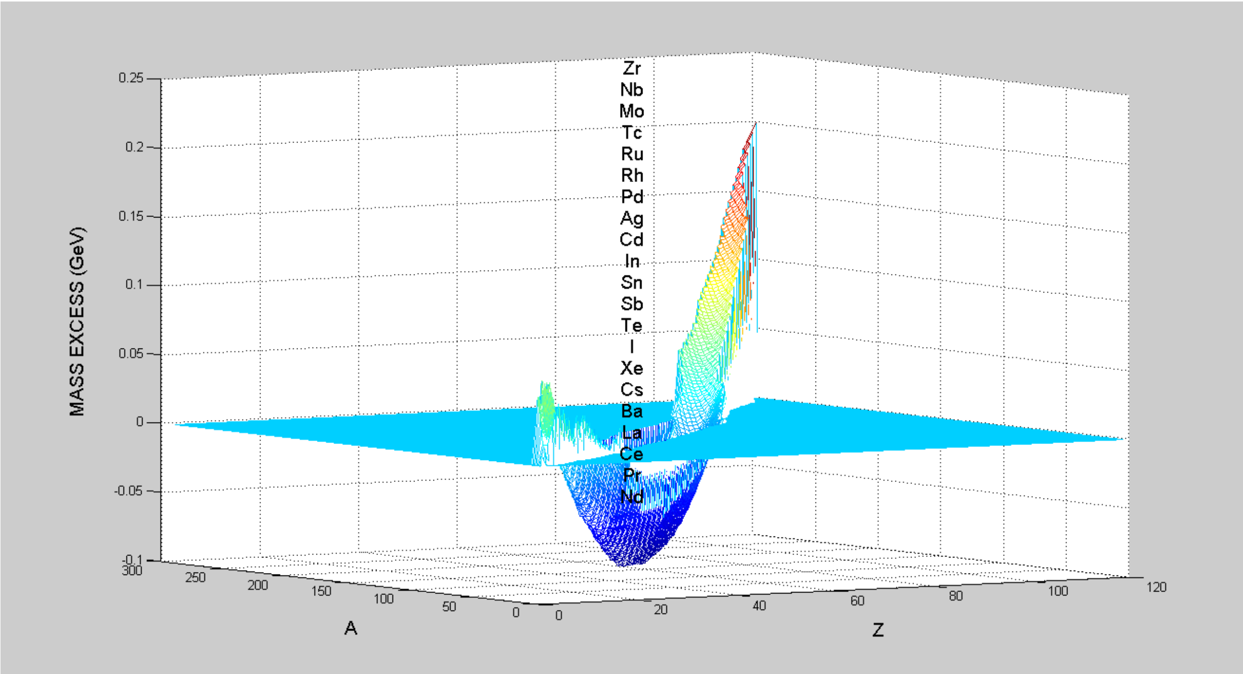

Figure 3: Mass excess depending on the charge Z and the number of nucleons A according to [19].

From the data presented in Fig. 3 that forms a mass excess above the plane (A, Z) looks like the ark, at the bottom of which are elements from zirconium to neodymium, whose names are listed in a vertical column. The binding energy depends on the atomic number Z and mass number A as follows [20]

$$E_b = a_1 A - a_2 A^{2/3} - a_3 Z(Z-1)A^{-1/3} - a_4 (N-Z)^2 A^{-1} + a_5 A^{-3/4} \quad (23)$$
$$a_1 = 14;\ a_2 = 13;\ a_3 = 0.585;\ a_4 = 19.3;\ a_5 = 33\delta(A, N, Z)$$

Here are displayed current values of the coefficients in MeV obtained on the basis of data [19]. In this expression, the function $\delta(A, N, Z)$ is defined as follows:

$\delta = 1$, for even Z, N;

$\delta = -1$, for odd Z, N;

$\delta = 0$, for odd A.

Equation (23) is a simple semi-empirical formula, proposed by Weiszacker about 1935.

Using the developed model of the interaction of protons with a scalar field, we can exclude neutrons from the description of the nuclei structure, treating them as special states of the hydrogen atom. Then the nucleus of an atom in its structure will be identical to the atom itself. It allows justifying the nuclear shell model [1-3].

Thus, we assume that the atom consists of protons A, interacting with a scalar field, which shields the N protons in accordance with expression (10), creating the nucleus. The rest of Z protons form the electron shell of an atom. The expression for the energy of the protons in this model can be written as

$$E_{bp} = A(m_p + m_e) - (ME + Am_u) \tag{24}$$

Note that in nuclear physics is widely used by the upper bound of the binding energy, which is obtained from equation (24) by replacing the total mass of a proton and an electron on the mass of the neutron, ie,

$$E_{bn} = Am_n - (ME + Am_u) \tag{25}$$

The objective is to obtain an expression of the binding energy of the protons from the general theoretical models developed above. To find this expression, we note that the parameter E in equation (17) can be both real and complex values, which correspond to states with finite lifetime. Solving the first equation (17) with respect to energy, we find

$$E = \frac{-SbPu \pm i\sqrt{-(SbPu)^2 + (Sb+1)(S - P^2 + SbP^2u^2)}}{(Sb+1)} \tag{26}$$

Note that for most nuclides the decay time is large enough, so it can be assumed that the imaginary part of the right-hand side of equation (26) is a small value. Hence we find that for these states the following relation between the parameters is true

$$P^2 \approx \frac{S(Sb+1)}{1 + Sb(1-u^2)} \tag{27}$$

Substituting the momentum expression in equation (26), we find

$$E = \frac{S^{3/2}bu}{\sqrt{(Sb+1)(1+Sb(1-u^2))}} \tag{28}$$

Thus, we have established a link between the energy of the state and parameters of the magnetic interaction in the special state that are characterized by a finite lifetime (radioactive nuclei). Interestingly, these conditions depend on the magnetic charge, which appears in equations (3) - (4). Since, according to our hypothesis, the nucleus consists of protons, we have a simple equation for the metric parameter ratio

$$\frac{\varepsilon}{k} = \frac{A^2 e^2}{A m_p c^2} = \frac{A e^2}{m_p c^2}$$

Hence we find the dependence of energy on the number of nucleons

$$E/A = \frac{S^{3/2} b_0 A u}{\sqrt{(S b_0 A^2 + 1)(1 + S b_0 A^2 (1 - u^2))}} \tag{29}$$

It is indicated $b_0 = (2\alpha m_e / m_p (1 - 2a))^2$. For the best matching expression (29) with data [19] set $\sqrt{S} = 293; S b_0 = 0.003; u = 0.999$ - see Fig. 4.

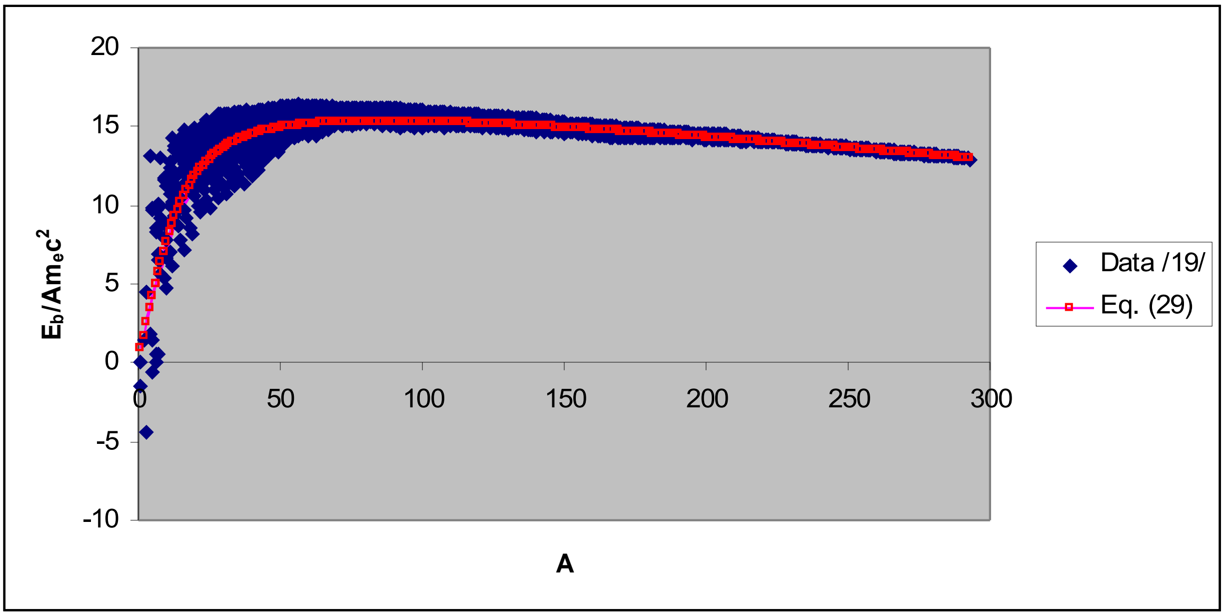

Figure 4: Binding energy per nucleon as a function of mass number according to [19] and calculated according to equation (29).

Equation (29) can approximately describe the binding energy of nucleons as function of mass number A for all nuclides. But for light nuclei, there is considerable disagreement with experiment, same as in the case of calculations with eq. (23). This is due, apparently, that the structure of light nuclei is strongly dependent on the details of the interaction. In general, the parameters S, b_0, u are constant for the entire set of nuclides only, but it is not true for particular nuclei with given N and Z.

In this regard, we note that in theory [4, 21] action in the five-dimensional space can be written as $\Sigma_5 = mcx^5 + \Sigma(x^1, x^2, x^3, x^4)$. Consequently, the wave vector in the fifth dimension corresponds to the mass and the normalized vector - the number of unit mass. The calculated value $\sqrt{S} = 293$ for the curve in Fig. 4, probably corresponds to the element with the highest mass number [293]Ei according to [19].

The average value of magnetic charge $u = 0.999$ indicates a high degree of correlation of the nucleons in the nucleus. The resulting value of the interaction parameter $b_0 = (2\alpha m_e / m_p(1 - 2a))^2 \approx 3.5 \cdot 10^{-8}$ allows us to determine the average value of the angular moment $l = \pm a \approx \pm 0.478744$. Further refinement of the model can be connected on the one hand, using the exact expression (26), and, on the other hand, the use of hypotheses about the behavior of the interaction parameters. In particular, it is necessary to take into account the effects of electromagnetic interactions and the distinction between protons and neutrons, ie dependence of energy on the nuclear charge Z.

The excited states

Consider the solutions (8) in the case when one can neglect the influence of gravity, $\lambda_1 \approx -\lambda_2 \approx 1$ but $\lambda = 1 + g_1^2(1 - u^2) \neq 1$. Under these conditions, equation (8) reduces to

$$-\frac{\omega^2}{c^2}\psi - \left(\psi_{rr} + \frac{1}{r}\psi_r - \frac{l^2}{r^2}\psi - k_z^2\psi\right) - \lambda k_\rho^2\psi + 2g^1 c^{-1}\omega k_\rho\psi - 2g^z k_z k_\rho\psi = 0 \tag{30}$$

In general, the solution of equation (30) can be represented in the form of power series, as in the analogous problem of excited states of the relativistic hydrogen atom [15-16]

$$\psi = \frac{\exp(-\tilde{r})}{\tilde{r}^a}\sum_{j=0}^{n} c_j \tilde{r}^j \tag{31}$$

It is indicated $\tilde{r} = r / r_n$. Substituting (31) in equation (30), we find

$$(a^2 - l^2 + \kappa_u)\sum_{j=0}^{n} c_j \tilde{r}^{j-2} + (2a - 1 + \kappa_g r_n)\sum_{j=0}^{n} c_j \tilde{r}^{j-1} +$$
$$(1 - k_z^2 r_n^2 + K^2 r_n^2)\sum_{j=0}^{n} c_j \tilde{r}^j - \sum_{j=0}^{n} j c_j \tilde{r}^{j-1} - 2a\sum_{j=0}^{n} j c_j \tilde{r}^{j-2} +$$
$$\sum_{j=0}^{n} c_j j(j-1)\tilde{r}^{j-2} = 0$$

It is indicated $\kappa_u = (1-u^2)k_\rho^2\varepsilon^2/k^2$. Hence, equating coefficients of like powers $\tilde{r} = r/r_n$, we obtain the equations relating the parameters of the model in the case of excited states

$$a = \sqrt{l^2 - \kappa_u}, \quad r_n = \frac{n+1-2a}{\kappa_g}, \quad \frac{1}{r_n^2} - k_z^2 + k_\rho^2 + \frac{\omega^2}{c^2} = 0 \quad (32)$$

Note that equation (32) apparently does not differ from those examined above equations (12). In particular, expressions (26) - (29) are true, in which should be put

$$\mathrm{Sb} = SA^2 b_{nl} = \frac{4SA^2(\alpha m_e/m_p)^2}{\left(n+1-2\sqrt{l^2-(1-u^2)SA^2(\alpha m_e/m_p)^2}\right)^2} \quad (33)$$

Note that if $n+1-2|l| = 0$ all the excited states have a common resonance level at which the interaction parameter (33) takes the value

$$\mathrm{Sb} \approx \frac{2(n+1)}{(1-u^2)^2 SA^2(\alpha m_e/m_p)^2} >> 1 \quad (34)$$

We simplify the expression (26) and (29), taking (34) into account, as a result we find

$$E \approx \frac{S^{1/2}u}{\sqrt{1-u^2}}, P^2 \approx \frac{S}{1-u^2} \quad (35)$$

Using the mean values calculated from data in Fig. 4, we find that the condition (35) has the total binding energy about 3345.4 MeV. At this state we have the following relations between the momentum and energy

$$E = -uP, \ P^2 - E^2 = S \quad (36)$$

Let us compare the obtained value of the binding energy to the kinetic energy of the nucleons in the nucleus, considering the neutrons and protons as a mixture of two kinds of Fermi gases [20]

$$U_t = c_t \frac{N^{5/3} + Z^{5/3}}{A^{2/3}} \quad (37)$$

The coefficient in this equation has to be agreed with the formula (23), in which the kinetic energy of the nucleons is described by the fourth term. Typically, matching is achieved by expanding the expression (37) in powers of small parameter $D = (N-Z)/A$. Hence we find $a_4 = 2^{-2/3}5c_t/9$, and therefore $c_t \approx 55.15 MeV$.

Table 4 presents the nuclides for which the kinetic energy of nucleons, calculated by the formula (37), 1% different from the average energy of the resonant state (35). Comparing the data of Table 4 with the data in Fig. 3, we find that the nuclides have kinetic energy comparable to the energy of the resonant states correspond to the elements with a minimum value of excess mass, which are collected at the bottom of the "ark."

Table 4: Nuclides having the kinetic energy of the nucleons different by 1% from the resonance states average energy of 3345.4 MeV.

A	N	Z	EL	U_t, MeV	E_{bp}, MeV	E_b, MeV
97	62	35	Br	3515,253	741,6834	790,162
97	61	36	Kr	3494,447	754,9474	802,675
98	62	36	Kr	3538,004	759,1163	807,618
98	61	37	Rb	3518,213	768,542	816,2639
98	60	38	Sr	3500,019	780,966	827,9056
99	61	38	Sr	3542,597	783,795	831,5169
99	60	39	Y	3525,378	791,8102	838,7498
99	59	40	Zr	3509,734	799,3778	845,535
99	58	41	Nb	3495,662	803,9363	849,3111
100	60	40	Zr	3551,316	805,5026	852,4422
100	59	41	Nb	3536,608	808,8376	854,9948
100	58	42	Mo	3523,455	815,0826	860,4575
100	57	43	Tc	3511,856	814,9145	859,507
100	56	44	Ru	3501,807	818,1173	861,9274
100	55	45	Rh	3493,307	814,4825	857,5103
101	58	43	Tc	3551,77	822,5231	867,8979
101	57	44	Ru	3541,054	824,137	868,7295
101	56	45	Rh	3531,872	823,5953	867,4055
101	55	46	Pd	3524,223	821,6153	864,6431
101	54	47	Ag	3518,105	817,4115	859,657
101	53	48	Cd	3513,518	811,9349	853,3981
101	52	49	In	3510,46	804,8013	845,471
101	51	50	Sn	3508,931	795,7473	835,674
102	55	47	Ag	3555,593	825,7412	868,769
102	54	48	Cd	3550,293	823,1542	865,3996
102	53	49	In	3546,508	814,1858	855,6488
102	52	50	Sn	3544,237	808,4058	849,0865

In analyzing the data in Table 4 the question arises, how small in magnitude the binding energy of the nucleons can hold them together with so much of their kinetic energy? Usually this question is ignored, because the derivation of the semi empirical equation (23) does not take into account all the kinetic energy, but only part that depends on a parameter $D = (N - Z)/A$. This theory allows us to answer this question. Indeed, the energy of the resonant state (35) can be regarded not only as a constant, which characterizes the entire set of nuclides, but also as a parameter that can be selected to compensate for the excess kinetic energy of the nucleons for each nuclide.

Finally, consider the case of the interaction of the electric charge of the nucleus with a scalar field in the absence of magnetic interaction. Putting $u = 0$ in equation (26), we find

$$E = \pm\sqrt{\frac{P^2 - S}{1 + Sb}}, \mathrm{Sb} = \frac{4SA^2(\alpha m_e / m_p)^2}{\left(n + 1 - 2\sqrt{l^2 - SA^2(\alpha m_e / m_p)^2}\right)^2} \quad (38)$$

The excited state (38) have a resonance level $n+1-2|l|=0$ at which the interaction parameter becomes large in magnitude,

$$\mathrm{Sb} \approx \frac{2(n+1)m_p^2}{SA^2\alpha^2 m_e^2} >> 1 \tag{39}$$

In this case, the expression of energy takes the form

$$E_n \approx \pm A\frac{\alpha m_e}{m_p}\sqrt{\frac{S(P^2-S)}{2(1+n)}} \tag{40}$$

Equation (40) has an extreme condition $S = P^2/2$ we then find the extreme value of energy

$$E_n \approx \pm \frac{A\alpha m_e}{m_p}\frac{P^2}{2}\sqrt{\frac{1}{2(1+n)}} = \pm \frac{AS\alpha m_e}{m_p}\sqrt{\frac{2}{1+n}} \tag{41}$$

For a typical value $S = 293^2$, which was found for data in Fig. 4, we have the estimate for the ground level $E_0 / A = 0{,}246562\ \mathrm{MeV}$, which obviously corresponds to the hydrino states [7, 15-16].

Consequently, the interaction of electric charges of protons with a scalar field alone cannot create a binding energy sufficient for the formation of nuclei in the absence of a magnetic interaction.

Our main result is that we obtained model, describing the collective effects of interaction of nucleons in nucleus. In this model, nucleus consists of protons only, interacting with a scalar field. The strong interaction can be explained as effect of a magnetic charge. As a basis for constructing the model we used a special metric in the five-dimensional space [4, 18, 22], which describes the combined effect of gravity and electromagnetism on the motion of matter. Finally, we note that the above model of the atomic nucleus can be useful in calculating the binding energy of nucleons in nucleus, which is currently carried out mainly by semi-empirical models.

References

1. Maria Goeppert-Mayer. On Closed Shells in Nuclei DOE Technical Report, Phys. Rev. Vol. 74; 1948. II DOE Technical Report, Phys. Rev. Vol. 75; 1949
2. Aage Bohr, Rotational Motion in Nuclei. Nobel Lecture, Stockholm, December 11, 1975.
3. Ben Mottelson, Elementary Modes of Excitation in the Nucleus, Nobel Lecture, Stockholm, December 11, 1975.
4. Трунев А.П. Фундаментальные взаимодействия в теории Калуцы-Клейна// Научный журнал КубГАУ. – Краснодар: КубГАУ, 2011. – №07(71). С. 502 – 527. – Режим доступа: http://ej.kubagro.ru/2011/07/pdf/39.pdf
5. Alexander Trunev. Neutron structure in Kaluza-Klein theory// Chaos and Correlation, №12, Dec. 15, 2011.

6. Alexander Trunev. Electron structure in Kaluza-Klein theory //Chaos and Correlation, №12, Dec. 7, 2011 http://chaosandcorrelation.org/Chaos/CR12_2011.pdf
7. Trunev A. P. Electron structure, hydrino and cold fusion//Chaos and Correlation, № 11, Nov. 25, 2011, http://chaosandcorrelation.org/Chaos/CR11_2011.pdf
8. Alexander P. Trunev. Lorentz quantum electrodynamics// Научный журнал КубГАУ [Электронный ресурс]. – Краснодар: КубГАУ, 2012. – №01(75). С. 1075 – 1090. – Режим доступа: http://ej.kubagro.ru/2012/01/pdf/83.pdf
9. Alexander Trunev. Neutron decay in the classic and quantum mechanics//Chaos and Correlation, April 30, 2011, http://chaosandcorrelation.org/Chaos/CR_4_2011.pdf
10. Mohr P.J., Taylor B.N., Newell D.B. CODATA recommended values of the fundamental physical constants// Reviews of Modern Physics 80: 633–730. 2006.
11. Richard P. Feynman. The Theory of Fundamental Processes. Addison Wesley. ISBN 0-8053-2507-7. (1961).
12. Christman J. The Weak Interaction. Physnet. Michigan State University, 2001. http://physnet2.pa.msu.edu/home/modules/pdf_modules/m281.pdf
13. Hunting the Quark Gluon Plasma. RESULTS FROM THE FIRST 3 YEARS AT RHIC. ASSESSMENTS BY THE EXPERIMENTAL COLLABORATIONS. Relativistic Heavy Ion Collider (RHIC). BNL -73847-2005, April 18, 2005.
14. John Arrington, Kees de Jager and Charles F. Perdrisat. Nucleon Form Factors-A Jefferson Lab Perspective// http://arxiv.org/PS_cache/arxiv/pdf/1102/1102.2463v1.pdf
15. Naudts, Jan (5 August 2005). On the hydrino state of the relativistic hydrogen atom. arXiv:physics/0507193.
16. Dombey, Norman (8 August 2006). "The hydrino and other unlikely states". Physics Letters A 360: 62. arXiv:physics/0608095
17. Mills, Randell L. (June 2008). "The Grand Unified Theory of Classical Physics" (DjVu). Blacklight Power. http://www.blacklightpower.com/theory/bookdownload.shtml
18. V. Dzhunushaliev. Wormhole solutions in 5D Kaluza-Klein theory as string-like objects// arXiv:gr-qc/0405017v1
19. National Nuclear Data Center Web Site, http://www.nndc.bnl.gov/
20. Marselo Alonso, Edward J. Finn. Fundamental University Physics. III Quantum and Statistical Physics. – Addison-Wesley Publishing Company, 1975.
21. Ю. Б. Румер. Исследования по 5-оптике. – М., Гостехиздат,1956. 152 с.
22. Kaluza, Theodor. Zum Unitätsproblem in der Physik. Sitzungsber. Preuss. Akad. Wiss. *Berlin. (Math. Phys.)* **1921**: 966–972.