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Хаос и фазовые переходы в атомных ядрах

Chaos and phase transitions in atomic nuclei

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В работе рассмотрена модель хаотического поведения нуклонов в атомных ядрах, построенная на основе модели ядерных взаимодействий и статистики Ферми-Дирака. Показано, что в такой модели наблюдаются фазовые переходы, при которых энергия и химический потенциал образуют на плоскости замкнутые фигуры.

Ключевые слова: нейтрон, протон, ядро, ядерная оболочка, хаос, химический потенциал, энергия связи.
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The model of chaotic behavior of nucleons in nuclei, based on the model of nuclear interactions and the Fermi-Dirac statistics is discussed. It is shown that in this model there are phase transitions, and in the chemical potential and energy plane there are specific geometric figures.

Keywords: Binding Energy, Chaos, Chemical Potential, Neutron, Nuclei, Nuclei Shell, Proton.

It is known that the binding energy of nucleons in nuclei depends on the availability of a regular motion of protons and neutrons in the nuclear shells, and on the chaotic behavior of the nucleon, which introduces uncertainty in the measurement of the mass of the nuclides [1-3]. Models of chaotic behavior of the nucleon are based on an analogy with the chaos in classical dynamical systems, as well as on the concept of quantum chaos [4-5]. In [6] developed a model of the bifurcation of the binding energy in atomic nuclei, based on the theory of nuclear interactions [7] and on the generalized dynamics of the Verhulst-Ricker-Planck equation [8]. To derive the equations of the model using the relationship between the size of the nucleus, binding energy and the number of neutrons and protons, this relation can be represented as follows (see [6-7])

$$rE = B(A,Z) \tag{1}$$

Here $A = Z + N, N, Z$ are the number of nucleons, neutrons and protons, respectively. Using the experimental data [9] and the standard expression of the nuclear radius, reflecting the weak compressibility of nuclear matter, i.e. $r(A) = r_0 A^{1/3}$, we can define the left-hand side of equation (1). As a result, we find the radius of the core product of the energy due to the number of nucleons. For consistency with the data [9], we set

$$B/r_0 = a_0 + a_1 A + a_2 A^{4/3} + a_3 Z^2 + a_4 (N-Z)^2 A^{-2/3} \quad (2)$$

$$a_0 = -14438.078;\ a_1 = -15418.779;\ a_2 = 15181.734;$$
$$a_3 = -687.601;\ a_4 = -22502.817$$

Here are the values of the coefficients derived from the data [9] for the binding energy calculated relative to the carbon isotope ^{12}C. All the coefficients are given in keV. Note that expression (2) has a high degree of accuracy for all nuclides with the number of nucleons $A \geq 20$ - Fig. 1.

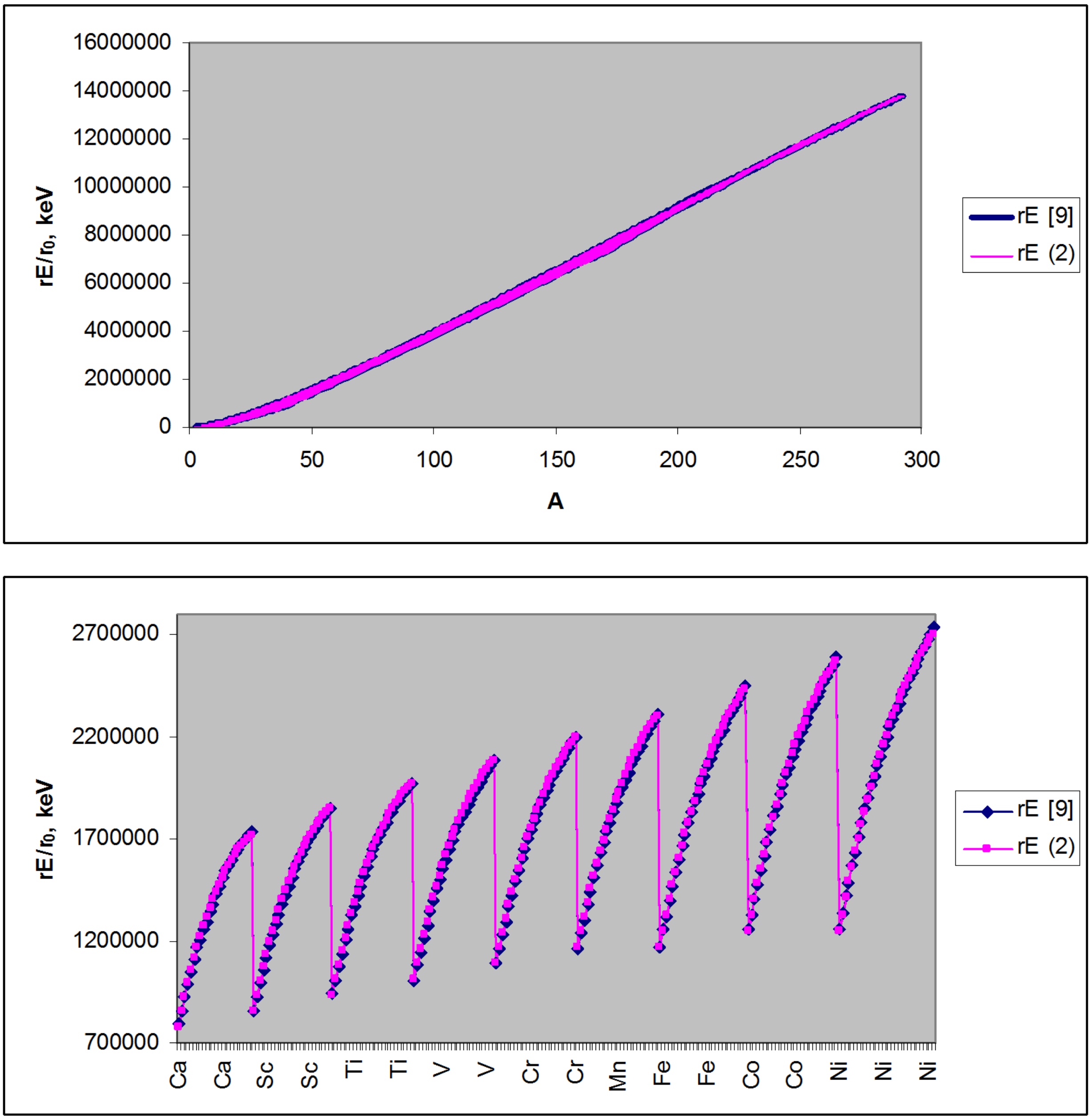

Figure 1: The dependence of the product of nuclei size and binding energy due to the number of nucleons (top) and for number of isotopes (bottom) calculated according to equation (2) and according to [9] is shown.

Construct on the basis of equation (1) a discrete model of the energy levels in nuclei as follows:

$$E_{A+1}E_A^2 = \frac{B_{A+1}B_A^2}{r_{A+1}r_A^2} = \frac{A}{4\pi r_A^3/3}\frac{4\pi r_A}{3r_{A+1}}\frac{B_{A+1}B_A}{A} \tag{3}$$

On the other hand, the density of nucleons in the nucleus can be related to energy using the Fermi-Dirac statistics, we have

$$n_A = \frac{A}{4\pi r_A^3/3} = \frac{g_Z Z/A}{e^{(E_Z-\mu_Z)/\theta}+1} + \frac{g_N N/A}{e^{(E_N-\mu_N)/\theta}+1} \tag{4}$$

Here g_i, E_i, μ_i, θ are the weight factors, energy and chemical potential of protons and neutrons, and the statistical temperature of the nucleon, respectively. Consider the results obtained in the simplified model under the condition of equality of chemical potentials and binding energies of the two kinds of nucleons:

$$\mu_N = \mu_Z = \mu_A = -\theta b, \quad E_Z = E_N = -E_A/A.$$

In this case, the model can be written as follows

$$(x_{A+1}+b)(x_A+b)^2 = \frac{K}{e^{-x_A}+1}$$
$$x_A = -\frac{E_A}{A\theta} - b, \quad K = \frac{4\pi g_A}{3\theta^3}\frac{B_A B_{A+1}}{A^4}(1+1/A)^{2/3} \tag{5}$$
$$g_A = g_N + g_Z$$

Model (5) differs from similar models developed in [6-7] in that it has no singularity at the point $x_A = 0$. For a fixed number of nucleons the first equation (5) can be regarded as a model of equilibrium in the system of nucleons at nonzero temperature [6]. In this case we have

$$x_{i+1} = \frac{K}{(x_i+b)^2(e^{-x_i}+1)} - b \tag{6}$$

The main properties of the model (6) coincide with the properties of the model discussed in [6-7]. In particular, the bifurcation diagram of model (6) has a characteristic form of "four rats", and reproduces the transition to chaotic behavior in the parameter values $b > \ln 137$ – Fig. 2.

Consider the two-dimensional generalization of the model (6), which occurs when the chemical potential deviation from the set value is proportional to the binding energy, we have:

$$x_{i+1} = \frac{K}{(x_i+b)^2(e^{-x_i}+1)} - b_1 + y_i, \quad y_{i+1} = \beta x_i \tag{7}$$

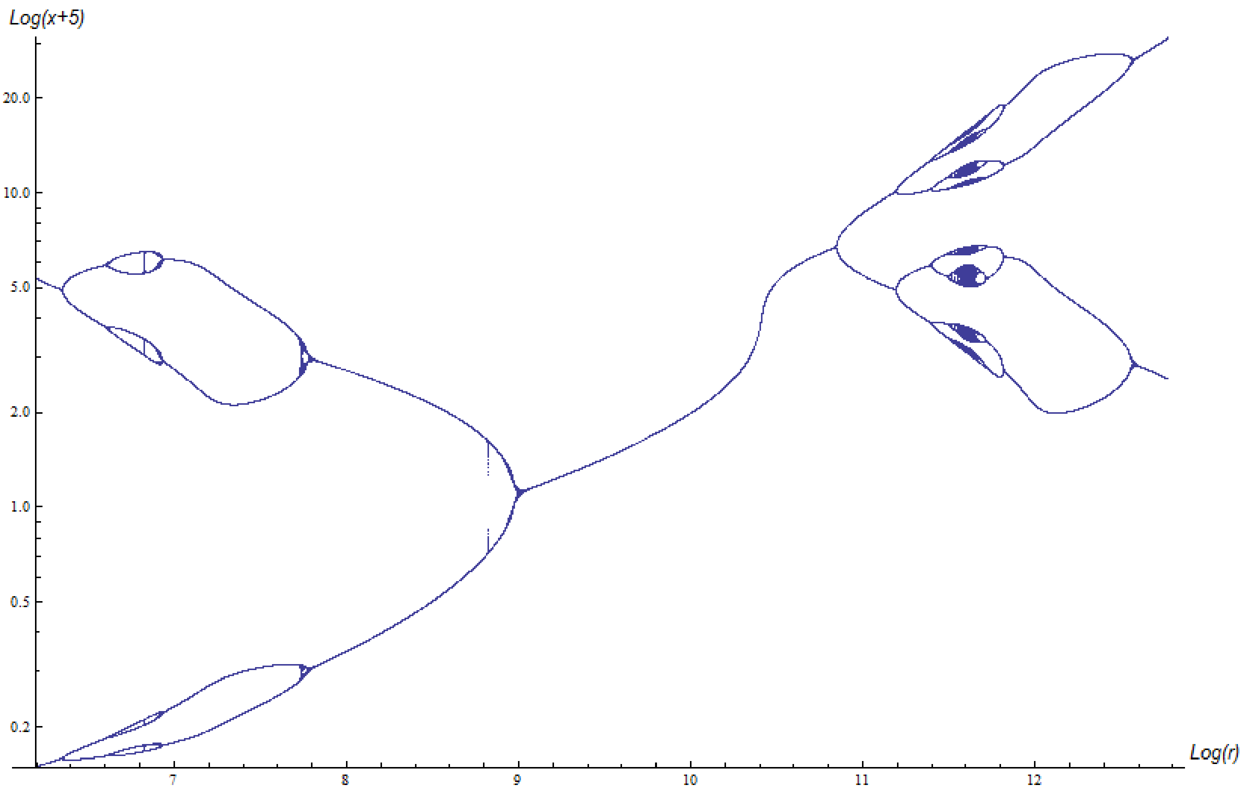

Figure 2: Bifurcation diagram "four rats", which contains specific zones of chaotic behavior. Calculations made with model (6) at $b = \ln 137.035999$.

Model (7) has a number of interesting properties. In the range $b_1 = b = \ln 137;\ \beta = -0.63;\ 800 \le K \le 4840$ model has a solution, similar to a strange attractor, as described in [10-11] and others - see Fig. 3. In the parameter range $b = \ln 137;\ b_1 = b/137;\ \beta = -1.0001;\ K > 0$ the solution have a form of some geometric figure, apparently indicating the phase transitions in the system of nucleons - Fig. 4. Finally, we consider the solution of equation (7) in the one dimension case, i.e. with parameters $\beta = 0;\ y_i = 0$, $b = \ln 137;\ b_1 = b/137$, for which the figures shown in Fig. 4 have been calculated. In this case, equation (6) and (7) differ only in the magnitude of the constant on the right side of these equations. Nevertheless, their bifurcation diagrams differ quite significantly - compare Fig. 3 and 5. For the solutions of equation (7) for the indicated values of the parameters of the bifurcation diagram has only two branches. In the vicinity of the bifurcation point there is thickening of the solutions that form the line spectrum of energy - Fig. 5.

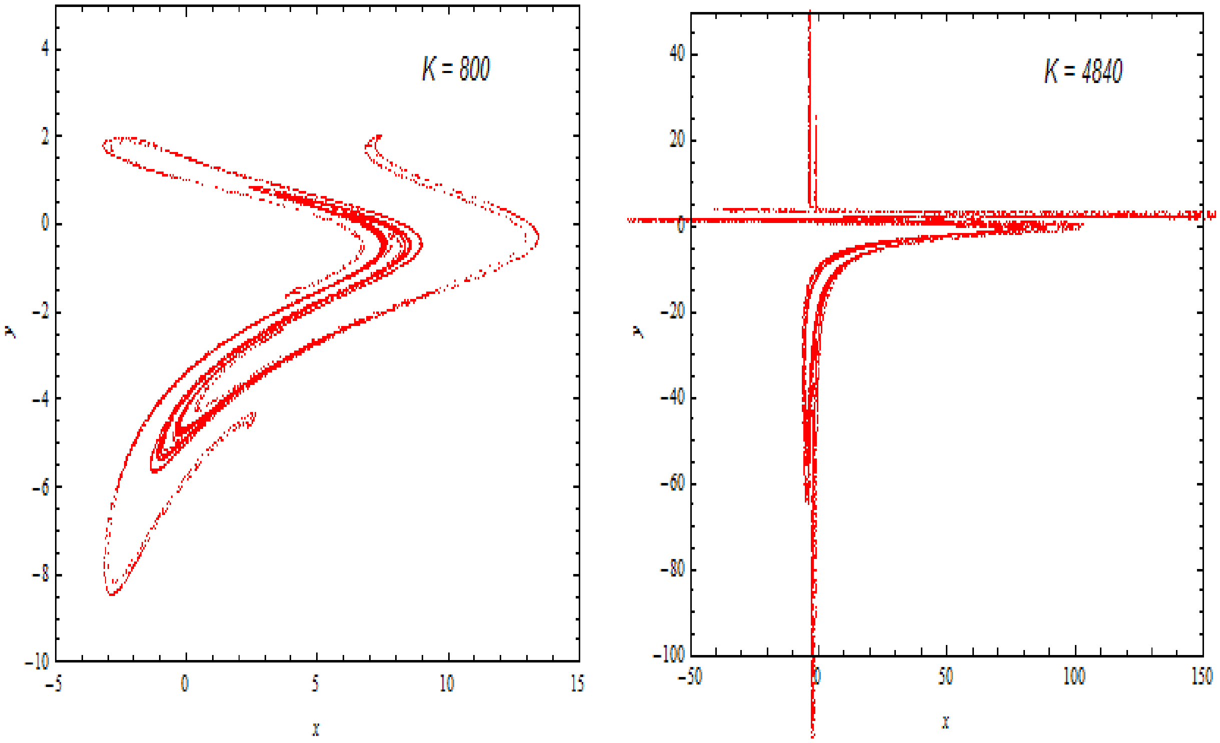

Figure 3: The strange attractor arising in the model (7) in the range of parameters $b_1 = b = \ln 137;\ \beta = -0.63;\ 800 \le K \le 4840$.

Thus, we have shown that the system of nucleons in nuclei at finite temperature phase transitions can be observed due to the mutual influence of changes in energy and chemical potential, as well as a line spectrum of energy and chaos that previously observed in the model [6].

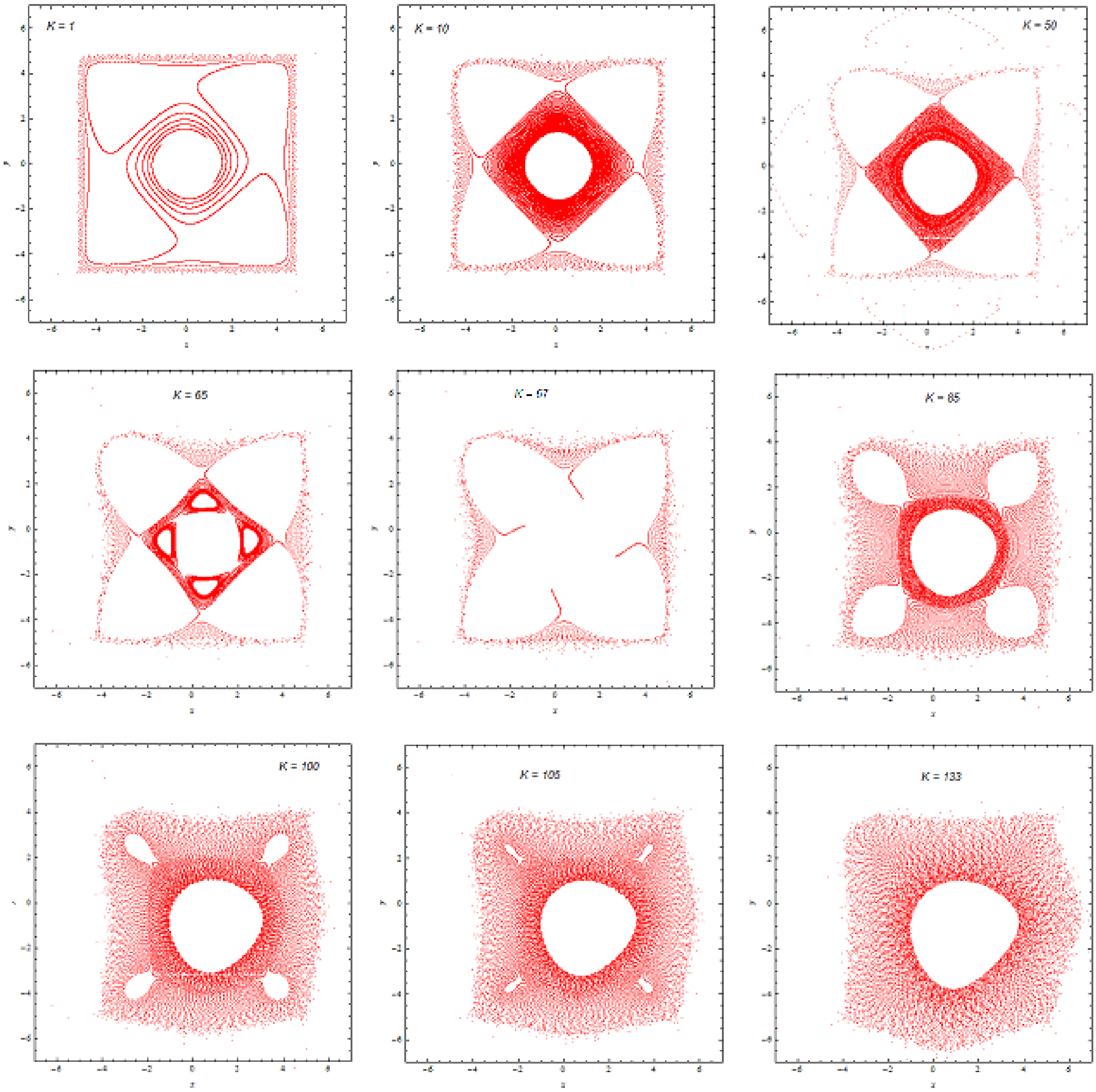

Figure 4: The characteristic shapes that form in the plane of the system (7), describing the energy and chemical potential of nucleons at a constant temperature in the range of parameters $b = \ln 137; b_1 = b/137; \beta = -1.0001; 1 \le K \le 200$.

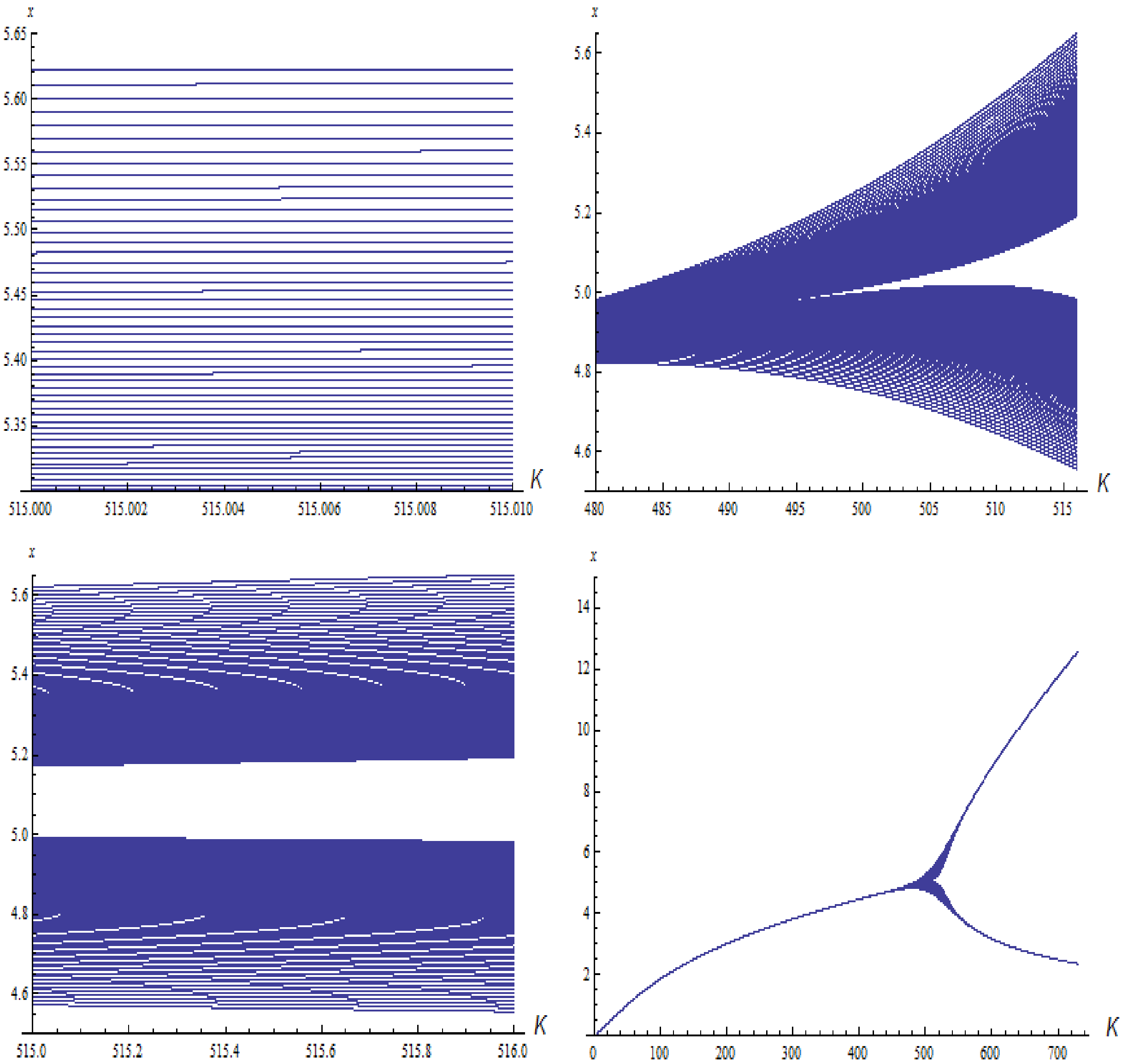

Figure 5: The bifurcation diagram with the line spectrum of energy. Calculations made with the model (7) at $\beta = 0; y_i = 0$ and at $b = \ln 137; b_1 = b/137$.

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