



**Моделирование массы
адронов на основе
скалярной модели глюоболов**

**Simulation of hadron masses
on the basis of the scalar
model of glueballs**

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В работе рассмотрена скалярная модель глюонного конденсата, в котором образуются глюболы. Показано, что масса известных адронов описывается с приемлемой точностью интегралом от плотности конденсата по объему глюбола.

In this paper we consider a scalar model of the gluon condensate, in which bubbles are formed - glueballs. It is shown that the mass of the known hadrons is described with acceptable accuracy by the integral of the condensate density in terms of the glueball.

Ключевые слова: адрон, глюонный конденсат, глюбол, масса, скалярное поле.

Keywords: gluon condensate, glueball, hadron, mass, scalar fields.

According to modern ideas hadrons consist of quarks interacting through a vector gauge bosons - gluons. Quantum chromodynamics (QCD), describing this kind of interaction is extremely complex theory, so that models of elementary particles, based on QCD, which are widely used to simplify and various numerical methods. Glueball is a hypothetical particle predicted by QCD [1]. It is assumed that the glueball is only from the gluon condensate. According to the calculations made in the framework of lattice QCD [2], a scalar particle of this type has a mass about 1730 MeV.

In this paper we used a scalar model of the gluon condensate, developed in [3-4] and others. This model, in the notation of [4] has the form

$$\begin{aligned}\partial_{\mu}\partial^{\mu}\phi &= -\phi\left[\chi^2 + \lambda_1(\phi^2 - \phi_{\infty}^2)\right] \\ \partial_{\mu}\partial^{\mu}\chi &= -\chi\left[\phi^2 + \lambda_2(\chi^2 - \chi_{\infty}^2)\right]\end{aligned}\quad (1)$$

Here, ϕ, χ the scalar fields describe the distribution of condensate; λ_1, λ_2 - the model parameters; $\phi_{\infty}, \chi_{\infty}$ - the eigenvalues of the problem. In the case of spherical symmetry of the system of equations (1) reduces to

$$\begin{aligned} x\phi'' + 2\phi' &= ax\phi \left[\chi^2 + \lambda_1(\phi^2 - \phi_\infty^2) \right] \\ x\chi'' + 2\chi' &= ax\chi \left[\phi^2 + \lambda_2(\chi^2 - \chi_\infty^2) \right] \end{aligned} \tag{2}$$

Here we have introduced a dimensionless variable $x = ra^{-1/2}$. The boundary conditions for the system of equations (2) have the form:

$$\begin{aligned} \phi(0) &= 1, \quad \phi'(0) = 0, \\ \chi(0) &= \chi_0, \quad \chi'(0) = 0. \end{aligned} \tag{3}$$

The system of equations (2) with boundary conditions (3) was solved using Wolfram Mathematica 8 [5] for the parameter values from [4]:

$$a = 1; \lambda_1 = 0.1; \lambda_2 = 1; \phi_\infty = 1.6171579; \chi_\infty = 1.49273856.$$

The results of calculations of the functions ϕ, χ are shown in Figure 1.

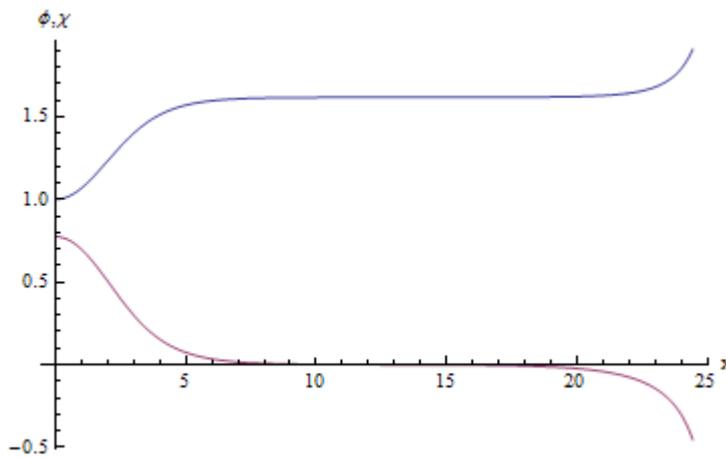


Figure 1: Glueball parameters, calculated according to [4].

As can be seen from the data shown in Figure 1, glueball is a spherical formation with a density dependent on coordinates. In theory [3-4], the density of the condensate is described by an effective Lagrangian

$$G = -L_{eff} = \langle H_i^A H^{Ai} \rangle - \langle E_i^A E^{Ai} \rangle \tag{4}$$

Here E_i^A, H_i^A - chromoelectric and chromomagnetic field respectively. Expression of the condensate density as a function of the distribution of scalar fields is given by [4]

$$G = -\frac{1}{2}(\phi'^2 + \chi'^2) + \frac{\lambda_1}{4}(\phi^2 - \phi_\infty^2)^2 + \frac{\lambda_2}{4}(\chi^2 - \chi_\infty^2)^2 - \frac{\lambda_2}{4}\chi_\infty^4 - \frac{1}{2}\phi^2\chi^2 \tag{5}$$

In the particular case of the subgroup of SU (2) the expression (5) reduces to

$$G_{SU(2)} = -\frac{1}{2}\phi'^2 + \frac{\lambda_1}{4}(\phi^2 - \phi_\infty^2)^2 \quad (6)$$

Expressions (5) - (6), together with the solutions of (2) - (3) were used to simulate the mass of hadrons - Figure 2-3. We assume that hadrons are composed of a central core - glueball, surrounded by a fur coat from the quark and gluon fields. For each hadron glueball has a certain radius and mass of the glueball is determined by the integral of a linear combination of functions (5) and (6). In addition, the glueball mass contributes to the surface tension due to the finite size of the glueball. Thus, the mass of the glueball is determined according to

$$m = 4\pi a^{3/2} \int_0^{x_0} (G + bG_{SU(2)} + k\rho/x)x^2 dx \quad (7)$$

We have considered two models of density $\rho = \phi^2 + \chi^2$ - Fig. 2, and $\rho = 1$ - Fig. 3. Both models have the same accuracy in comparison with the mass of hadrons, which is apparently explained by the behavior of functions preserving the constant value in a wide range of variation of the radial coordinate. In addition, a separate functional mass has been studied in the case of SU (2) condensate:

$$m = 4\pi a^{3/2} \int_0^{x_0} (G_{SU(2)} + k\rho/x)x^2 dx \quad (8)$$

Model (7) - (8) was tested for the entire set of hadrons - Fig. 2-3. Assume that the mass of a single hadron is proportional to the mass of its glueball, therefore, we have (9)By changing the parameters of the model, we can achieve agreement dependencies (7) - (8) with tabular data hadron masses. To solve this problem, we used the built-in Wolfram Mathematica 8 [5] a table of elementary particles with the parameters ParticleData ["Hadron", "Mass"]. Removed from the table data sheet, which adds a number of zero particle - 175 for the model (7) and 100 for model (8). These data allow us to combine the origin, in which the mass of the hadron and glueball mass are linearly related (9). The data for hadrons normalized to the maximal element - $m_Y = 11\,019$ MeV. Further adjustment of model parameters is carried out - a, b, h, k for the model (7) and a, h, k for model (8). The parameters are stored in all the glueball calculations, namely:

$$\lambda_1 = 0.1; \lambda_2 = 1; \phi_\infty = 1.6171579; \chi_\infty = 1.49273856.$$

As a result, we obtained the following values of the parameters of the model (7):

$$m_H / m_Y = hm / 4\pi ,$$

$$\rho = \phi^2 + \chi^2 : a = 0.0003815; b = 1.792; h = 0.3665, k = 0.0237; \quad (10)$$

$$\rho = 1 : \quad a = 0.0003815; b = 1.792; h = 0.3665; k = 0.061$$

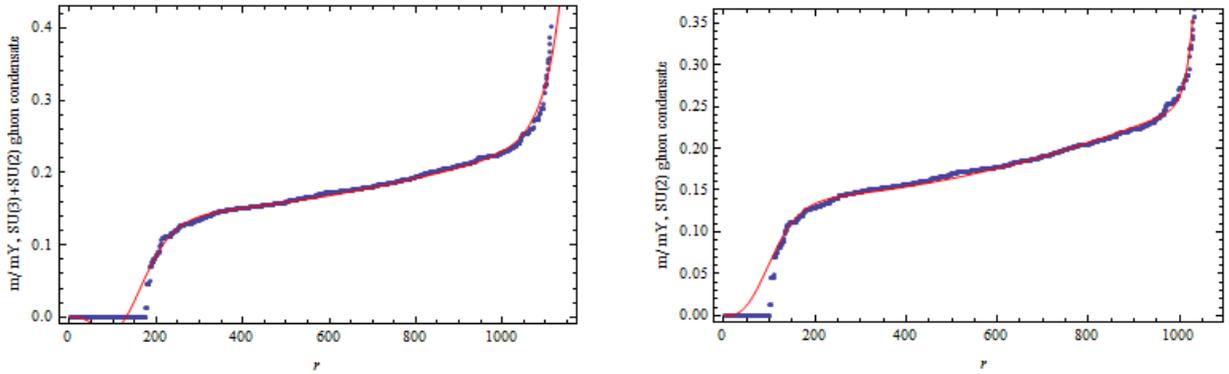


Figure 2: Comparison of hadron masses with the mass of the glueball, calculated from equations (7) - (8). Parameters of the model (7): $a = 0.0003815$; $b = 1.792$; $k = 0.0237$; $h = 0.3665$. The model parameters (8): $a = 0.000536$; $k = 0.0164$; $h = 0.414$; $m_Y = 11\,019$ MeV. In

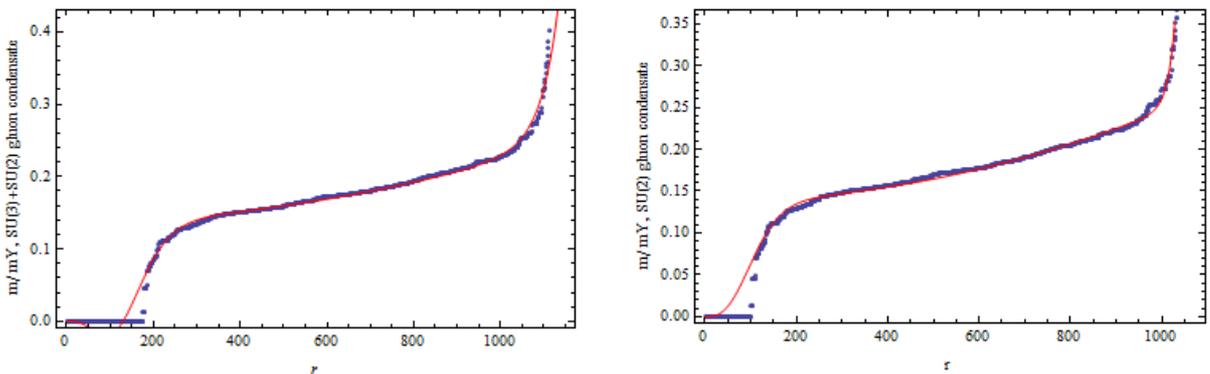


Figure 4: Comparison of hadron masses with the mass of the glueball, calculated from equations (7) - (8). Parameters of the model (7): $a = 0.0003815$; $b = 1.792$; $k = 0.061$; $h = 0.3665$. The model parameters (8): $a = 0.000536$; $k = 0.042$; $h = 0.414$; $m_Y = 11\,019$ MeV.

Comparison of hadron masses with the mass of the glueball, as calculated by the model (7) with data (10) is given in Fig. 2-3. A satisfactory agreement between the calculated and experimental data starts with the masses ρ - meson component of 775.5 MeV and ends at the mass ψ - meson component of the 4421 MeV. For hadrons at less and large mass a linear model (9) is not satisfied.

For model (8) yielded the following values

$$\begin{aligned} m_H / m_Y &= hm / 4\pi , \\ \rho &= \phi^2 + \chi^2 : a = 0.000536; h = 0.414, k = 0.0164; \\ \rho &= 1 : a = 0.000536; h = 0.414; k = 0.042 \end{aligned} \quad (11)$$

Note that the difference in the accuracy of the description of the experimental data between the models (7) and (8) is the nominal, but the model (8) contains one parameter less. On the other hand, the difference in density of the models used to simulate the surface energy is also the nominal and reduced only to a redefinition of the parameter k , while maintaining the values of other model parameters, as follows from expressions (10) - (11).

Thus, we have shown that the linear model (9) connecting the hadron mass to the mass of the central nucleus - glueball is performed for a large part of the hadron mass in the range of 775.5 MeV to 4421 MeV - about 922 particles from a total of 973. This supports the accepted model of the structure of elementary particles, in which it is assumed that hadrons contain a central core of the glueball and the surrounding fields of quarks and gluons.

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