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**Моделирование энергии возбужденных состояний атомных ядер на основе скалярной модели глюоболов**

**Simulation of the atomic nuclei excited states on the basis of the scalar model of glueballs**

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В работе рассмотрена скалярная модель глюонного конденсата, в котором образуются глюболы. Показано, что энергия возбужденных состояний ядер описывается с приемлемой точностью интегралом от плотности конденсата по объему глюбола.

In this paper we consider a scalar model of the gluon condensate, in which bubbles are formed - glueballs. It is shown that the energy of the nuclei excited states is described with acceptable accuracy by the integral of the condensate density in terms of the glueball.

Ключевые слова: атомное ядро, возбужденные состояния, адрон, глюонный конденсат, глюбол, масса, скалярное поле.

Keywords: gluon condensate, excited states, glueball, hadron, mass, nuclei, scalar fields.

According to modern views the atomic nucleus consists of nucleons - protons and neutrons, which in turn consist of quarks interacting through a vector gauge bosons - gluons. Quantum chromodynamics (QCD), describing this kind of interaction is extremely complex theory, so the core models of elementary particles, based on QCD, which are widely used to simplify and various numerical methods. Glueball is a hypothetical particle predicted by QCD [1]. It is assumed that the glueball is only from the gluon condensate. According to the calculations made in the framework of lattice QCD [2], a scalar particle of this type has a mass about 1730 MeV.

In [3] have modeled the mass of hadrons based on the model glueball [4], using Wolfram Mathematica 8 [5]. It was shown that the mass of all the known hadrons with acceptable accuracy by the integral of the density of the condensate in terms of the glueball. In this paper, a model glueball [4] is used to model the excited states of atomic nuclei. This model, in the notation of [4] has the form

$$\begin{aligned} \partial_{\mu} \partial^{\mu} \phi &= -\phi \left[ \chi^2 + \lambda_1 (\phi^2 - \phi_{\infty}^2) \right] \\ \partial_{\mu} \partial^{\mu} \chi &= -\chi \left[ \phi^2 + \lambda_2 (\chi^2 - \chi_{\infty}^2) \right] \end{aligned} \quad (1)$$

Here, the scalar fields  $\phi, \chi$  describe the distribution of condensate;  $\lambda_1, \lambda_2$  - the model parameters;  $\phi_\infty, \chi_\infty$  - the eigenvalues of the problem. In the case of spherical symmetry the system of equations (1) reduces to

$$\begin{aligned} x\phi'' + 2\phi' &= ax\phi[\chi^2 + \lambda_1(\phi^2 - \phi_\infty^2)] \\ x\chi'' + 2\chi' &= ax\chi[\phi^2 + \lambda_2(\chi^2 - \chi_\infty^2)] \end{aligned} \tag{2}$$

Here we have introduced a dimensionless variable  $x = ra^{-1/2}$ . The boundary conditions for the system of equations (2) have the form:

$$\begin{aligned} \phi(0) &= 1, \quad \phi'(0) = 0, \\ \chi(0) &= \chi_0, \quad \chi'(0) = 0. \end{aligned} \tag{3}$$

The system of equations (2) with boundary conditions (3) was solved using Wolfram Mathematica 8 [5] for the parameter values from [4]:

$$a = 1; \lambda_1 = 0.1; \lambda_2 = 1; \phi_\infty = 1.6171579; \chi_\infty = 1.49273856$$

The results of calculations of the functions  $\phi, \chi$  are shown in Figure 1.

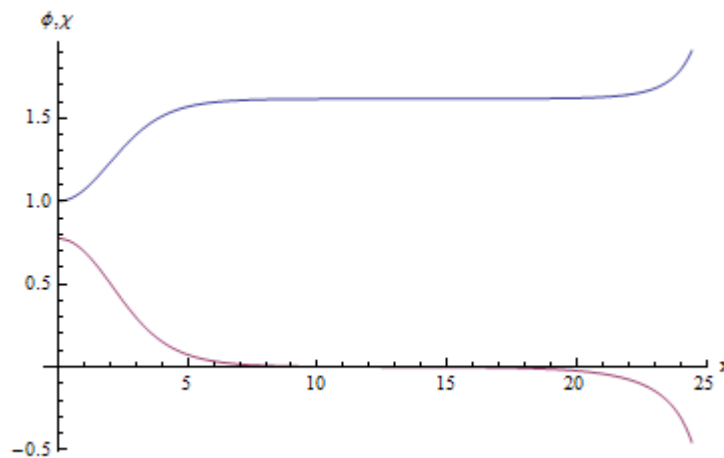


Figure 1: Glueball parameters, calculated according to [4]  $a = 1; \lambda_1 = 0.1; \lambda_2 = 1; \phi_\infty = 1.6171579; \chi_\infty = 1.49273856$ .

As can be seen from the data shown in Fig. 1, glueball is a spherical formation with a density dependent on coordinates. In theory [4, 6], the density of the condensate is described by an effective Lagrangian

$$G = -L_{eff} = \langle H_i^A H^{Ai} \rangle - \langle E_i^A E^{Ai} \rangle \tag{4}$$

Here  $E_i^A, H_i^A$  - chromoelectric and chromomagnetic field respectively. Expression of the condensate density as a function of the distribution of scalar fields has the form [4]

$$G = -\frac{1}{2}(\phi'^2 + \chi'^2) + \frac{\lambda_1}{4}(\phi^2 - \phi_\infty^2)^2 + \frac{\lambda_2}{4}(\chi^2 - \chi_\infty^2)^2 - \frac{\lambda_2}{4}\chi_\infty^4 - \frac{1}{2}\phi^2\chi^2 \quad (5)$$

In the particular case of the subgroup of SU (2) the expression (5) reduces to

$$G_{SU(2)} = -\frac{1}{2}\phi'^2 + \frac{\lambda_1}{4}(\phi^2 - \phi_\infty^2)^2 \quad (6)$$

Expressions (5) - (6), together with the solutions of (2) - (3) were used to model the hadron masses [3] – Figure 2. It is assumed that hadrons are composed of a central core - glueball, surrounded by a coat of quark and gluon fields. For each hadron glueball has a certain radius and mass of the glueball is determined by the integral of a linear combination of functions (5) and (6). In addition, the glueball mass contributes to the surface tension due to the finite size of the glueball. Thus, the mass of the glueball is determined according to

$$m = 4\pi a^{3/2} \int_0^{x_0} (G + bG_{SU(2)} + k\rho/x)x^2 dx \quad (7)$$

It was considered two models of density  $\rho = \phi^2 + \chi^2$  - Figure. 2, and  $\rho = 1$ . As it was shown the models have the same accuracy in comparison with the mass of hadrons, which is apparently explained by the behavior of functions  $\phi, \chi$  preserving the constant value in a wide range of variation of the radial coordinate. In addition, a separate functional mass has been studied in the case of SU (2) condensate:

$$m = 4\pi a^{3/2} \int_0^{x_0} (G_{SU(2)} + k\rho/x)x^2 dx \quad (8)$$

Assuming that the mass of a single hadron is proportional to the mass of its glueball, we have

$$m_H = Hm \quad (9)$$

By changing the parameters of the model, we can achieve agreement dependencies (7) - (8) with tabular data hadron masses. To solve this problem, use the built in Wolfram Mathematica 8 [5] a table of elementary particles with the parameters ParticleData[“Hadron”, “Mass”].

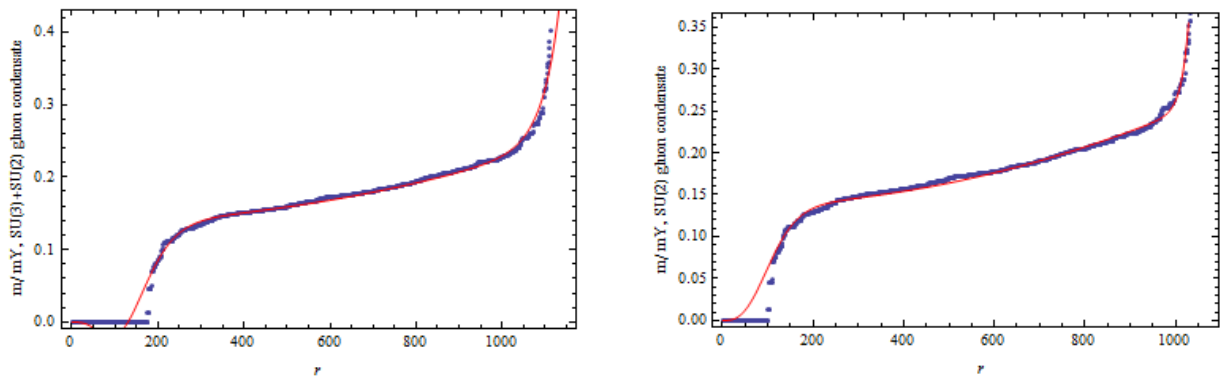


Figure 2: Comparison of hadron masses with the mass of the glueball, calculated from equations (7) - (8) at  $\rho = \phi^2 + \chi^2$ . The model parameters (7):  $a = 0.0003815$ ;  $b = 1.792$ ;  $k = 0.0237$ ;  $h=0.3665$ . The model parameters (8):  $a=0.000536$ ;  $k = 0.0164$ ;  $h = 0.414$ ;  $mY = 11019$  M $\bar{e}B$ .

To model the energy of the excited states of nuclei - Fig. 3-4, we use the model (7) - (9) and a built-in Wolfram Mathematica 8 [5] data of isotopes with appropriate parameters. For example, data on the left in Figure 3 is called with parameters `IsotopeData["Ni58", "ExcitedStateEnergies"]`.

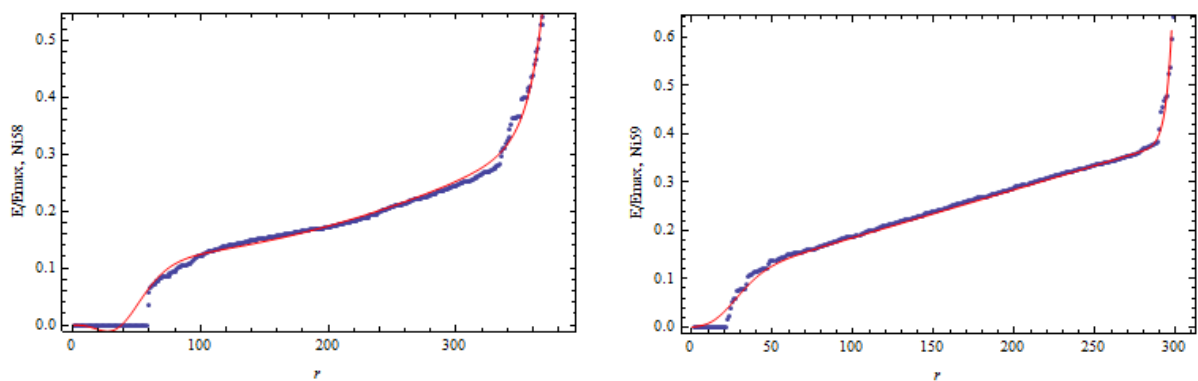


Fig. 3. Comparison of the excited states energy of nickel isotopes with glueball energy, calculated from equations (7) - (8) at  $\rho = \phi^2 + \chi^2$ . The model parameters (7) for Ni58:  $k = 0.01906$ ;  $h=0.2698$ ;  $a = 0.003756$ ;  $b = 1.94$ . The model parameters (7) for Ni59:  $a = 0.0068$ ;  $k = 2.09$ ;  $h = 0.3235$ .

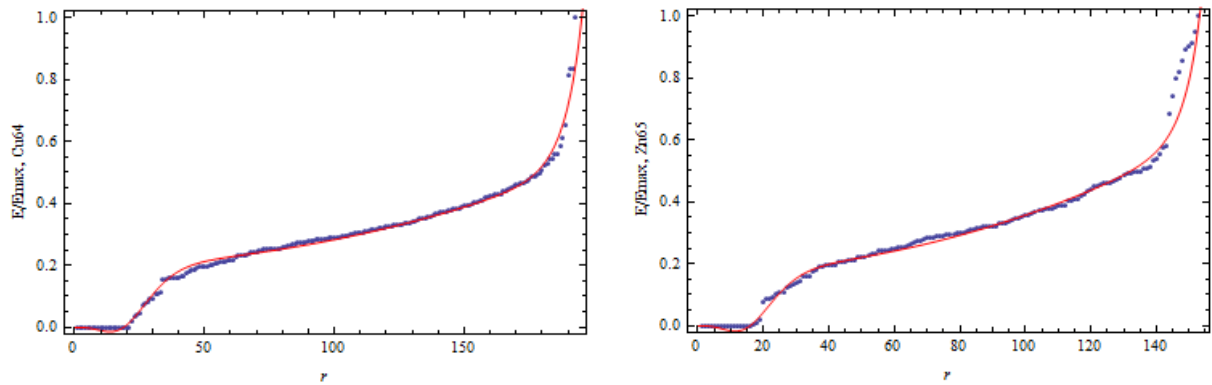


Рис. 4. Comparison of the excited states energy of the copper and zinc isotopes with glueball energy, calculated from equation (7) at  $\rho = \phi^2 + \chi^2$ . The model parameters (7) for Cu64:  $k = 0.0092$ ;  $h = 0.44$ ;  $a = 0.01356$ ;  $b = 2.085$ . The model parameters (7) for Zn65:  $a = 0.02168$ ;  $b = 1.962$ ;  $k = 0.00984$ ;  $h = 0.44$ .

From the table of isotopes extracted data sheet, which adds a number of null states. These data allow us to combine the origin, in which the energy of the excited state and the glueball mass are linearly related (9). The data for the energy of the excited states are normalized to the maximal element. Further, by fitting the model parameters -  $a, b, h, k$  for model (7) and  $a, h, k$  for model (8).

For each isotope selected own parameters, which indicate there are an individual scenario glueball in each case. For example, for the isotope Ni59 surface tension parameter is used not in the form  $k/x$ , as for Ni58, but in the form  $k/x^2$ , which is apparently due to the influence of angular momentum, which is not included in the model (2).

Thus, we have shown that the linear model (9) connecting the hadron mass to the mass of the central nucleus - glueball extends to the excited states of atomic nuclei. In this case the glueball, probably, should be considered as a bubble formed in the quantum condensate when the nucleus is excited, the same way as the formation of pores in solids and cavitation bubbles in a liquid under tension.

Finally, we note that for the simulation of the linear stage of the glueball in the excitation of atomic nuclei, it is possible to use one, for example, first equation (1), supplemented by terms that take into account the oscillations of the bubble. This kind of model of a quantum harmonic oscillator is widely used in the simulation of nuclear shells [7-8]. In [9-10] for the nuclear shell model was used scalar wave equation in five-dimensional space, which is four-dimensional space, is reduced to the first equation (1). In this sense, the model [4, 6] (and the model of the

excited states of nuclei developed above) is an obvious nonlinear generalization of linear models of the nuclear shells, consistent with the structure of hadrons.

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