Chaos and Correlation
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Фрактально-кластерная теория распределения ресурсов в социо-экономических системах

Fractal-Cluster Theory of Resource Distribution in Socio-Economic Systems

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Представлены основы фрактально-кластерной теории, в том числе фрактально-кластерной корреляции (FCC), динамические уравнения фрактально-кластерной системы эволюции и критерии для комплексного управления системами. Анализ экономических систем управления осуществляется на основе синтеза термодинамики И. Пригожина и фрактально-кластерной корреляции.
В статье показана связь между фрактально-кластерным и традиционным экономическим анализом экономических систем. Особенность фрактально-кластерной теории заключается в возможности оптимизации распределения бюджета в условиях неопределенности и прогнозировании возможных кризисных тенденций в экономическом развитии системы заранее.
Целью данного исследования является разработка аналитического аппарата для устойчивого анализа распределения ресурсов в сложных самоорганизующихся системах, который основан на пригожинской термодинамической структуре и фрактально-кластерной корреляции В.П. Бурдакова. Оптимизация распределения ресурсов особенно важна для таких социально-экономических систем, как фундаментальные науки, образование, социальная сфера. Информация о стоимости производства продукта в таких системах является недостаточной. В связи с этим, классические модели межотраслевого баланса (статическая и динамическая) В. Леонтьева не работают.
Анализ подтвердил, что наиболее эффективное управлением распределения ресурсов осуществляется посредством последовательности Фибоначчи с помощью нового математического аппарата на основе золотого сечения [A.Stakhov, 2005-2006].

Ключевые слова: фрактально-кластерная теория, фрактально-кластерная корреляция, последовательность Фибоначчи, золотое сечение.

The fundamentals of fractal-cluster theory, including fractal-cluster correlation (FCC), the dynamic equations of the fractal-cluster system evolution and the criteria for the complex systems management are presented. The analysis of economic systems management is performed on the basis of the synthesis of I.Prigogine’s thermodynamics of structure foundations and fractal-cluster correlation.

The article shows the correlation between the fractal-cluster and the traditional economic analysis for economic systems. The singularity of the fractal-cluster theory lies in the possibility to optimize the budget distribution under undetermined conditions and to predict possible crisis tendencies in the economic system development in advance.

The purpose of the given research is to work out the analytical apparatus for sustained resource distribution analysis of a complex self-organizing system, which is based on I.Prigogine’s thermodynamics structure and V.P Burdakov’s fractal-cluster correlations. The optimization of resource distribution is especially important for such socioeconomic systems as fundamental science, education, social sphere. The information about the cost of a thing produced in such systems is insufficient. Due to this fact, V. Leontief’s classical models of input-output balance (static and dynamic) do not work.
The analysis confirmed that the most efficient management of resource distribution is carried out by the Fibonacci sequence with the help of the new mathematical apparatus based on the Golden Section [A.Stakhov, 2005-2006].

Keywords: fractal-cluster theory, including fractal-cluster correlation, Fibonacci sequence, Golden Section.

1. Introduction

The basis of management, analysis of management efficiency and operation of self-organizing system (SS) which is assumed in the present research are the thermodynamic method and the fractal-cluster correlation [1,2].

Long years of statistical research [1] make it possible to prove that any system: technical, biological, which passed an evolutionary way of development; the machine-man systems always have five major clusters. They are: energy (S_e), transport (S_{tr}), technological (S_t), ecological (S_e) and informational (S_i) clusters which have certain (ideal) values, expressed as a percentage or fraction of the whole for extensive system parameter (time, money, weight, etc.). For the energy cluster, this value is 38%, transport - 27%, ecological - 16%, technological - 13%, informational - 6%. (Fig.1.1.) Each of the five clusters has five subclusters, for example: in the energy cluster - the energy support for energy system itself, energy support for transport, ecology, technology, information science, etc. The other subclusters are respectively divided into five subclusters at the next level. To analyze the functioning of biological, technical and anthropogenic systems the second or third level of FCC is generally enough. Such clustering [1] allowed to evaluate the complex system functioning. However, the theory or mathematical models based on FCC have not been developed so far.

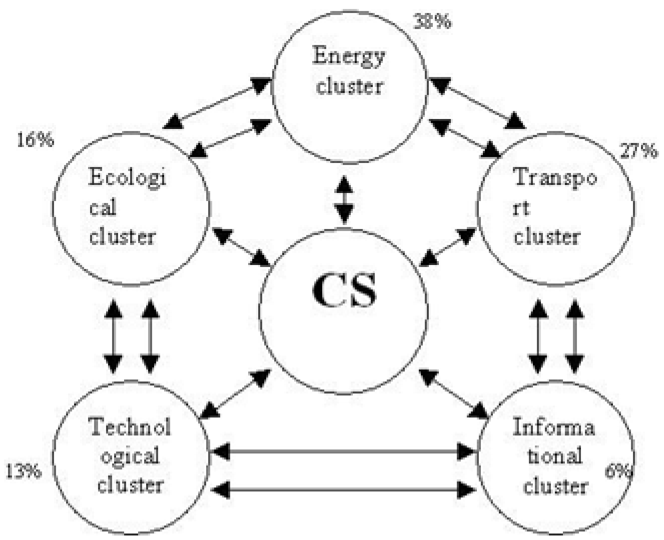


Fig. 1.1. A complex system as a fractal (one-dimensional case)

There are five direct and indirect proofs of the validity of a complex system fractal-cluster structuring which are based on:

1) experimental data plus mathematical statistics;

2) thermodynamics (thermodynamics of steady states) and theoretical aerohydrodynamics (Volov V.T.[3]);

3) theory of random processes; (Rusanov O.V. [4]);

4) theory of numbers;

5) algebra. ("The Golden Section" $x = \frac{1+\sqrt{5}}{2} = 1 + H_0$, where H_0 = 0,618. [5, 6, 7]

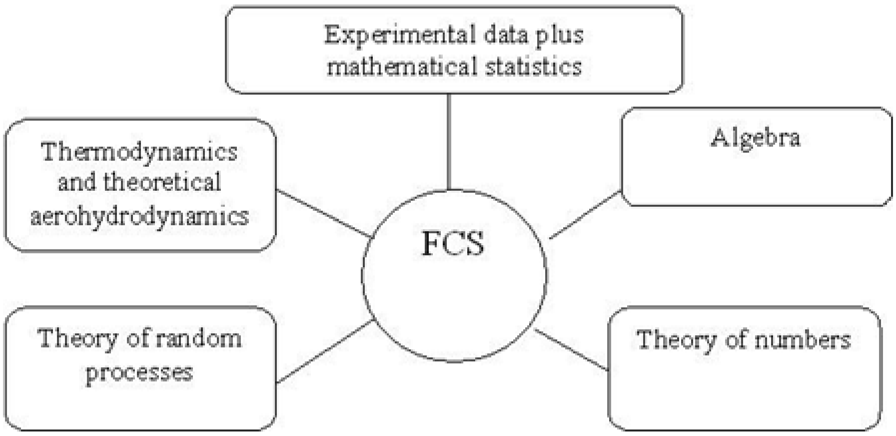


Fig. 1.2. Fractal-cluster structuring of a complex system

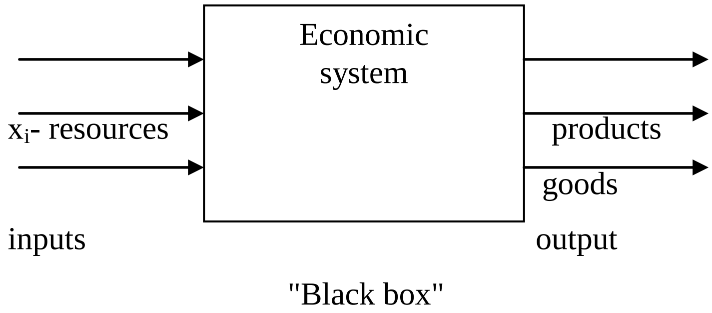
The aim of this study is to work out a theory of structural control of complex systems management based on the synthesis of economics, FCC and the non-equilibrium thermodynamics. By complex self-organizing system only economic systems (ES) of micro-and meso-levels are meant here.

Study of resource distribution in ES based on the fractal-cluster approach, show that management object is not decomposed, but is a "black box" (Fig. 1), which corresponds to the principles and methodology of thermodynamics. In the physical space the real object of economic management has the resources X_i, necessary for the functioning, and results of its activities (goods, products, services, etc.). The known economic and mathematical models (IO inputs-output), both static and dynamic (Leontiev's models [8]), describe the functioning of economic system in the physical space of economic variables (Fig. 1a).

This class of economic and mathematical models has several advantages over other models: obviousness, the relative simplicity. However, there are some drawbacks: the need for significant amounts of additional empirical information, practical impossibility to analyze the stability of resource distribution in the economic systems of micro-and meso-level in the long term.

Upon transfer from the physical space of economic variables in the five-dimensional fractal-cluster, space decomposition and classification of information about economic object resources is accomplished (Fig. 1, 2), i.e., the fractal-cluster structuration of the information about the resources needed to meet the EC needs (energy, transport, ecological, technological and informational). The universal thermodynamics device in its informational interpretation is convenient to use in this case. The laws and theorems of thermodynamics make it possible to analyze the stability of ES resource distribution, without additional empirical information. However, this class of models has its drawbacks: unconventional approach - in an explicit form, without additional empirical information, it is impossible to determine the criteria of ES activities (revenue, profit, profitability, etc.).

a)



b)

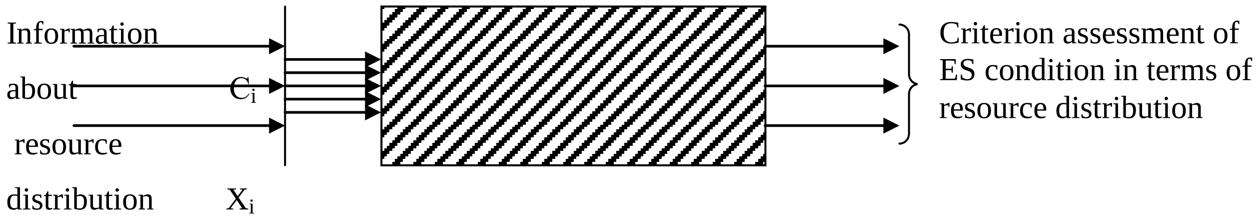


Fig. 1.3. Diagram representing the economic system in the traditional (a) and fractal - cluster (b) interpretation

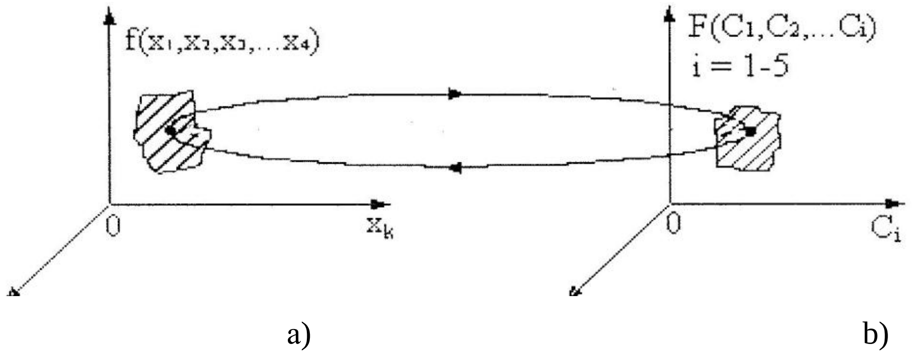


Fig. 1.4. The scheme of transfer from the physical space of economic variables into fractal-cluster space

Fractal-cluster criterion ES are defined in a nontrivial way by the construction of fractal-cluster model (FCM). In the fractal-cluster space (Fig. 2b), the cluster values $\{C_i\}$ and subclusters of any level is a positive value: $C_i > 0$.

Scope of permissible values of clusters (subclusters) changes is defined as follows:

$$R_C \in R_C^{add} = \begin{cases} 0 < C_i < a_i; \\ 0 < C_{ij} < b_{ij}; \\ 0 < C_{ij\ldots n} < d_{ij\ldots n}; \\ \sum_{i=1}^{5}\sum_{j=1}^{5}\sum_{m=1}^{5}\ldots\sum_{n=1}^{5}\overline{C}_{ijm\ldots n} = 1, \end{cases} \quad \text{where } a_i, b_{ij}, d_{ij\ldots n}>0,\ \overline{C}_{ijm\ldots n} = C_{ijm\ldots n}/C^{\Sigma}, \tag{1}$$

The cost of all resources structured in clusters is:

$$C_i = \sum_{k=1}^{n} r_{ik}\cdot\beta_k;\ \sum C_i = \sum_{i=1}^{5}\sum_{k=1}^{n} r_{ik}\cdot\beta_k = B\ \textit{or}\ \sum_{i=1}^{5}\vec{r}_i\cdot\vec{\beta} = \sum_{i=1}^{5}\sum_{k=1}^{n} r_{ik}\cdot\beta_k = B, \tag{2}$$

where C_i - the sum of different resources assigned to the i- cluster, $\vec{\beta}$ - cost ratio of the respective resource units, B - the system budget (consolidated budget), r_{ik} - share of the resources belonging to the i- cluster, C^{Σ} - the ES total resource in terms of value.

The proposed theory is based on:

1) axiom of the FCC universality (five-cluster structuring of the ES resource needs);
2) assumption that clusters $\{\overline{C}_i\}$ and subclusters of any level $\{\overline{C}_{ij}\}$, $\{\overline{C}_{ijk}\}$ ... $\{\overline{C}_{ij\ldots m}\}$ can not take zero value: $\overline{C}_i > 0$, $\overline{C}_{ij} > 0, \ldots \overline{C}_{ij\ldots m} > 0$;
3) assumption that the EC effective area in the physical space also corresponds to the effective functioning in the fractal-cluster space;
4) admission of the passive nature of system management (passive model). However, the of the control correction, i.e., feedback, takes place before a new stage of system transformation. Thus, here is a model with delayed feedback.

The task of resource distribution management can generally be formulated as follows: $\left|u-u^{stab}\right| \Rightarrow \min$, where u ^{stab} - stable resource distribution in the system, obtained on the basis of information - thermodynamic method (defined below).

The presented fractal-cluster theory includes:

1) V.P. Burdakov's fractal-cluster correlation (FCC) [1],

2) the dynamic equations for the fractal-cluster system evolution [2],

3) fractal-cluster criteria for system management efficiency,

4) analysis of complex self-organizing systems stability.

2. Dynamic equations FCC

Evolution of any ecomathermal n-level system from nonideal cluster-fractal state in ideal can be written as the system of equations [9, 10]:

$$\underbrace{\begin{matrix} r=1; \\ r=2; \\ r=3; \\ r=4; \\ \vdots \\ r=n \end{matrix} \left\{ \begin{array}{ll} \overline{C}_i(\bar{t}) = \overline{C}_i^0 + u_i(\varepsilon_i, \bar{t}) \cdot \overline{C}_i^0; & 3 \\ \overline{C}_{ij}(\bar{t}) = \overline{C}_{ij}^0 + u_{ij}(\varepsilon_{ij}, \bar{t}) \cdot \overline{C}_{ij}^0; & 4 \\ \overline{C}_{ijm}(\bar{t}) = \overline{C}_{ijm}^0 + u_{ijm}(\varepsilon_{ijm}, \bar{t}) \cdot \overline{C}_{ijm}^0; & 5 \\ \overline{C}_{ijm...n}(\bar{t}) = \overline{C}_{ijm...n}^0 + u_{ijm...n}(\varepsilon_{ijm...n}, \bar{t}) \cdot \overline{C}_{ijm...n}^0; & 6 \\ \sum_{i=1}^{5}\sum_{j=1}^{5}\sum_{m=1}^{5}...\sum_{n=1}^{5}\overline{C}_{ijm...n} = 1 & 7 \end{array} \right.}_{r\ \text{times}} \qquad (3\text{-}7)$$

at $0 \le \overline{t} \le 1$, where $\bar{t} = t/t_{end}$, t_{end} - end time of ES management,

where

$$\left\{ \begin{array}{ll} u_i = \left(\frac{C_i^{id}}{C_i^0} - 1\right) f_i(t) = \varepsilon_i f_i(t); & u_{ijm} = \left(\frac{C_{ijm}^{id}}{C_{ijm}^0} - 1\right) f_{ijm}(t) = \varepsilon_{ijm} f_{ijm}(t) \\ u_{ij} = \left(\frac{C_{ij}^{id}}{C_{ij}^0} - 1\right) f_{ij}(t) = \varepsilon_{ij} f_{ij}(t); & u_{ijm...n} = \left(\frac{C_{ijm...n}^{id}}{C_{ijm...n}^0} - 1\right) f_{ijm...n}(t) = \varepsilon_{ijm...n} f_{ijm...n}(t) \end{array} \right.$$

$$f_i(0) = f_{ijm...n}(0) = 0 \qquad (8)$$
$$f_i(1) = f_{ij}(1)...f_{ijm...n}(1) = 1$$

Here u_i, u_{ij}, u_{ijm}, $u_{ijm\,...\,n}$ - the controlling functions for clusters and subclusters of the first, second and (n-1) levels, $\overline{C}_{ij}^{id}$, $\overline{C}_{ijm}^{id}...\overline{C}_{ijm...n}^{id}$ - ideal relative values of subclusters of the first, second, ..., (n-1) levels, and $\overline{C}_{ij}^{01}$, $\overline{C}_{ijm}^{0},...,\overline{C}_{ijm..n}^{0}$ - initial relative values of the respective subclusters, f - a monotonous differentiable function $0 \le f \le 1$, whose form is either given or found from the additional stability conditions. Equation (7) is an analog of the conservation law for a fractal system.

3. Entropic-cluster method of the complex system management

Methods of management optimization proposed in [5, 6, 7] rely on intuitive or rigidly formalized concepts and analogies. In connection with the above said it is logical to formulate a criterion of matrix FCC management efficiency on the basis of fundamental principles of stable states thermodynamics.

Let us consider the matrix of FCC ideal states (two - dimensional case N=2) (Table 1).

Table 1: Table of ideal values

$\overline{C}_i$		$\overline{C}_{ij}$				
C_1	0,38	0,144	0,1026	0,0608	0,0494	0,0228
C_2	0,27	0,1026	0,0729	0,0432	0,0351	0,0162
C_3	0,16	0,0608	0,0432	0,0256	0,0208	0,096
C_4	0,13	0,0494	0,0351	0,208	0,0169	0,078
C_5	0,06	0,0228	0,0169	0,096	0,078	0,0036

The first row and first column resources of ideal matrix give quantitative information about the overall share of system energy resources, which is ~ 0.615, being the major determinant of the system functioning effectiveness:

$$\overline{C}_{\Sigma}^{\text{э}} = \sum_{j=1}^{5} \overline{C}_{1j} + \sum_{i=2}^{5} \overline{C}_{i1} \approx 0{,}615 \qquad (9)$$

This number is very close to the so-called "Golden section" of $H_o \approx$ 0,618 known from numerous publications as the basis of beauty and harmony in both natural and anthropogenic phenomena [6].

Fractal-cluster matrix (FCM) $\{\overline{C}_{ij}\}$ carries information about the energy state of the system studied. In connection with the above said it seems appropriate to use entropic approach to the fractal-cluster system management analysis.

Relationship between elements of FCM and the information entropy H allows us to find a criterion for FCM management for the purpose of optimal evolution from the nonideal state of the system (nonideal FCM) in perfect condition – (FCM ideal), at that, the sum of FCM elements of the first column and first row (9) goes into their ideal value, that is, the “Golden section” entropy value is achieved.

$$H = \sum_{j=1}^{5} \overline{C}_{1j} + \sum_{i=2}^{5} \overline{C}_{i1} \Rightarrow u_{ij} \Rightarrow \sum_{j=1}^{5} \overline{C}_{1j}^{\,ideal} + \sum_{i=2}^{5} \overline{C}_{i1}^{\,ideal} = H_0 \approx 0{,}618 . \qquad (10)$$

Thus, we introduce the hypothesis of FCC conditional entropy (or quasientropy) defining (10), based on a generalization of experimental data on the evolving system [1] and the FCM structure.

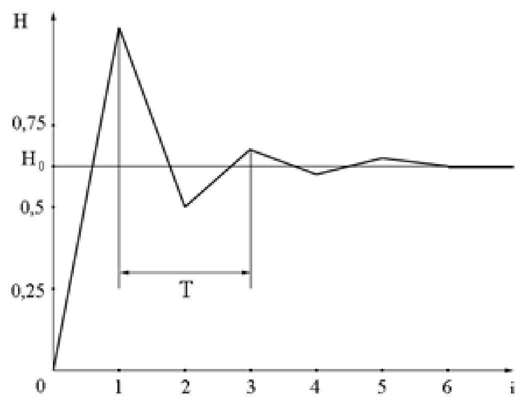
The proposed expression of fractal-cluster entropy is in terms of value the share of the system total resources used to meet its energy needs.

The structure of the FCM complex system is fractal: they are chains of repeating subclusters, self-similar in their structure. As is known fractal image is obtained by iterative processes. The elementary iterative process is the Fibonacci numbers.

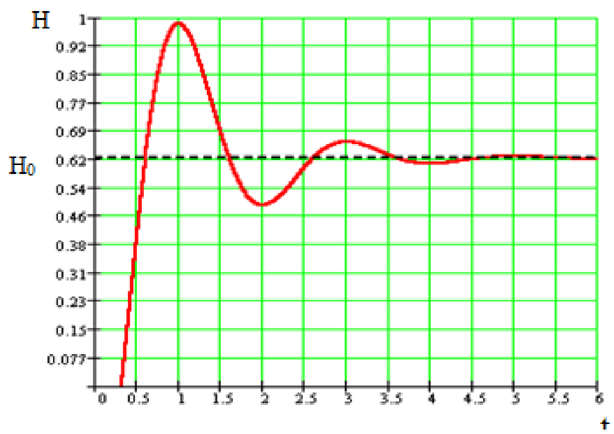
It turned out that the key to the fractal-cluster matrix (FCM) management is the famous Fibonacci numbers (0, 1, 1, 2, 3, 5, 8, 13, U_n ... U_{n+1}), in which each successive number is the sum of the previous two. A remarkable property of the Fibonacci numbers is that with numbers increasing the ratio of two adjacent numbers is asymmetrically close to the exact proportion of the "Golden section" which is the basis of beauty and harmony [11, 12] of both natural and numerous anthropogenic forms:

$$\lim_{n\to\infty}\frac{U_n}{U_{n+1}} = H_0 \approx 0{,}618 \qquad (11)$$

In connection with the above, there appeared a hypothesis about the optimal management of FCM by means of Fibonacci numbers. To control the FCM the approximation of Fibonacci numbers iterations [14] is used. Here iteration corresponds to time intervals that are multiple to the period of fluctuations attenuation, i.e. the approximation of the Fibonacci numbers iterations is a template for the matrix management $\{u_{ij}\}$ (Fig. 3.1.).



a)



б)

Fig.3.1a. Iteration of Fibonacci numbers:
i - iteration number, T = 2 - period
$H_0 \approx 0{,}618$ – "Golden section"

Fig. 3.1b. Approximation of Fibonacci numbers iterations

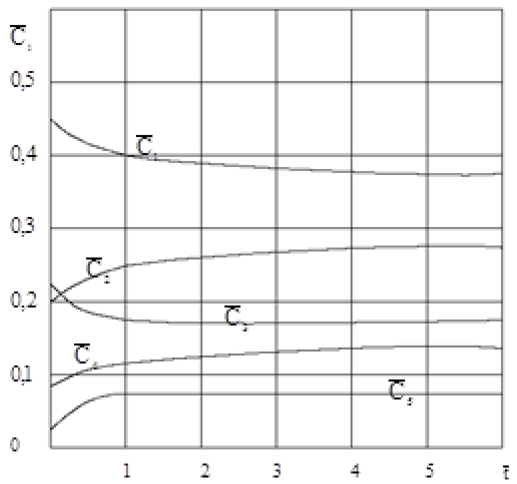


Fig. 3.2. Evolution of ES clusters according to Fibonacci numbers and sewing (14)

Managing matrix $\{u_{ij}\}$ at the same time of evolution beginning and end $\bar{t}^0_{ij} = \bar{t}^0_{ji} = \text{const}$ and $\bar{t}^{fin}_{ij} = \bar{t}^{fin}_{ji} = \text{const}_2$ takes the form:

$$u_{ij} = \left(\bar{C}^{id}_{ij} / \bar{C}^{0}_{ij} - 1\right) \cdot f\left(\bar{t} - \bar{t}_0\right) \tag{12}$$

Function $f\left(\bar{t} - \bar{t}_0\right)$ satisfies the conditions (8).

Approximation of the Fibonacci numbers iterations (Fig. 3.1.) gives the following expression:

$$f\left(\bar{t} - \bar{t}_0\right) = \frac{H}{H_0} = 1 + H_0 \cdot \exp\left(-\alpha\left(\bar{t} - \bar{t}_0\right)\right) \times \cos\left(\pi\left(\bar{t} - \bar{t}_0\right) + \varphi_0\right), \tag{13}$$

at the initial stage $\varphi_0 = 0$, $H_0 = 0{,}618$ $\alpha = 1{,}05$, $\bar{t}_0 = 1$.

Expression (13) fails to satisfy the initial conditions at $\bar{t} = t_0$. To satisfy the second boundary condition we introduce a new managing function u^* at the time interval from zero to some t:

$$u^*\left(\bar{t} - \bar{t}_0\right) = 1 - \exp\left(-\beta\left(\bar{t} - \bar{t}_0\right)\right), \tag{14}$$

and make sewing of solutions for $U\left(\bar{t} - \bar{t}_0\right)$:

$$\begin{cases} u_1^*(\bar{t}-\bar{t}_0)=f(\bar{t}-\bar{t}_0) \\ \left(u^*(\bar{t}-\bar{t}_0)\right)'=f'(\bar{t}-\bar{t}_0) \end{cases} \qquad \text{at } \bar{t}=\bar{t}_{sewing} \tag{15}$$

It is evident that the management $u^*(\bar{t}-\bar{t}_0)$ satisfies condition (8) at $\bar{t}=\bar{t}_0$. After simple transformations we get a system of transcendental equations:

$$\begin{cases} \beta=\alpha-\dfrac{\ln\left(-\cos\left[\pi\left(\bar{t}_{sewing}-\bar{t}_0\right)\right]\cdot H_0\right)}{\bar{t}_{sewing}-\bar{t}_0} \\ \ln\left[-\dfrac{1}{\cos\pi\left(\bar{t}_{sewing}-\bar{t}_0\right)\cdot H_0}\right]=tg\left(\pi\left(\bar{t}_{sewing}-\bar{t}_0\right)\right) \end{cases}, \tag{16}$$

where $\cos\left(\pi\left(\bar{t}_{sewing}-\bar{t}_0\right)\right)_0<0$

Numerically from equations (16) the value of t_{sewing} and the index β is determined. It turned out that $\bar{t}-\bar{t}_0\approx 1{,}19$; $\beta\cong 1{,}53$.

Fig.3.2. shows the evolution of clusters controlled by (13, 14).

As seen from Fig. 3a, there are stable (convex trajectory H, $d^2H/dt^2<0$) and unstable (concave trajectory H, $d^2H/dt^2>0$), corresponding to the obtained entropy-cluster solution based on the Fibonacci numbers. In this connection the hypothesis was suggested of structural waves of low intensity: the regimes where $\Delta H<<H_0$, $dP/dt>0$ is a mode of functional instability, which is an attribute of any developing self-organizing system. In the unstable regimes ($d^2H/dt^2>0$ and $\Delta H\sim H_{0)}$ there exists anomalous structural instability, which means serious crisis of structural processes in the ES.

4. Fractal-cluster criteria of a complex system effective resource distribution management

To assess the ES management in terms of the proposed approach it is necessary to develop static and dynamic criteria of its effective management. The above proposed criterion of conditional fractal-cluster entropy H (10) can be attributed to static criteria. In addition, the criteria of full effectiveness η^{Σ}, proposed in [1], can be treated as static criteria of the ES effectiveness. But these criteria are less sensitive.

Full effectiveness of the fractal-cluster system is determined, according to [1], as follows:

$$\eta^{\Sigma}=1-\sum_{i=1}^{5}\overline{C}_i^{ideal}\eta_i^{-1}\sqrt{\left(\overline{C}_i^{ideal}-\overline{C}_i\right)^2}, \tag{17}$$

where η_i - effectiveness of the i- cluster determined by:

$$\eta_i = 1 - \sum_{j=1}^{5} \overline{C}_{ij}^{ideal} \cdot \eta_{ij}^{n-1} \sqrt{\left(\overline{C}_{ij}^{ideal} - \overline{C}_{ij}\right)^2} \quad (18)$$

Calculation of subcluster effectiveness $\eta_{ij...m}$ (m-1)-level starts with the last (m-1)-level. Level number changes in the following way:

$$m-1 \le n \le 1 \quad (19)$$

But these criteria are less sensitive.

To determine a highly sensitive criterion of fractal-cluster matrix management effectiveness D_{eff} F.Hausdorff's approach was used. In contrast to the purely fractal structures, fractal-cluster n-dimensional matrix $FCM^{(n)}$ differ substantially from the geometrical fractal structures, as the quantitative distribution in subclusters of any level may differ from the ideal distribution and thus the system quality changes. However, redistribution in clusters and subclusters of any level obey the laws of conservation (7).

Therefore the following algorithm to determine the highly sensitive criterion of fractal-cluster n-dimensional matrix $FCM^{(n)}$ effectiveness is proposed.

Fractal-cluster criterion of management effectiveness (D-criterion) is determined by the formula:

$$D_{eff} = \frac{\log_5 \sum_{i=1}^{5} \sum_{j=1}^{5} \sum_{K=1}^{5} ... \sum_{m=1}^{5} \delta^*_{ijK...m}}{\log_5 N}, \quad (20)$$

where FCM is a m-dimensional matrix, N – number of clusters.

Besides, unlike F.Hausdorff's dimension, FC – dimension can take negative values.

In formula (17) values $\delta_i^*, \delta_{ij}^*, \delta_{ijK...m}^*$ are calculated by the relations:

$$\delta^*_{ijK...m} = 1 - \sqrt{\left(\frac{\overline{C}_{ijK...m}^{ideal}}{\overline{C}_{ijk...m}} - 1\right)^2} \quad (21)$$

Unlike entropy H and full effectiveness η_Σ, criteria D_{ef} and χ are very sensitive indicators of varying FCM values (Fig.4.1.) The area outside the boundary (hatched area) is a non-functional state of the system controlling. Figure 4.1. shows that in the sector of negative values of D_{ef} and χ, at the boundary of the system, we have the destruction of the space continuity where intensive FCM parameters can vary. This phenomenon can be interpreted as the boundary where the irreversible damage of the economic system management appears.

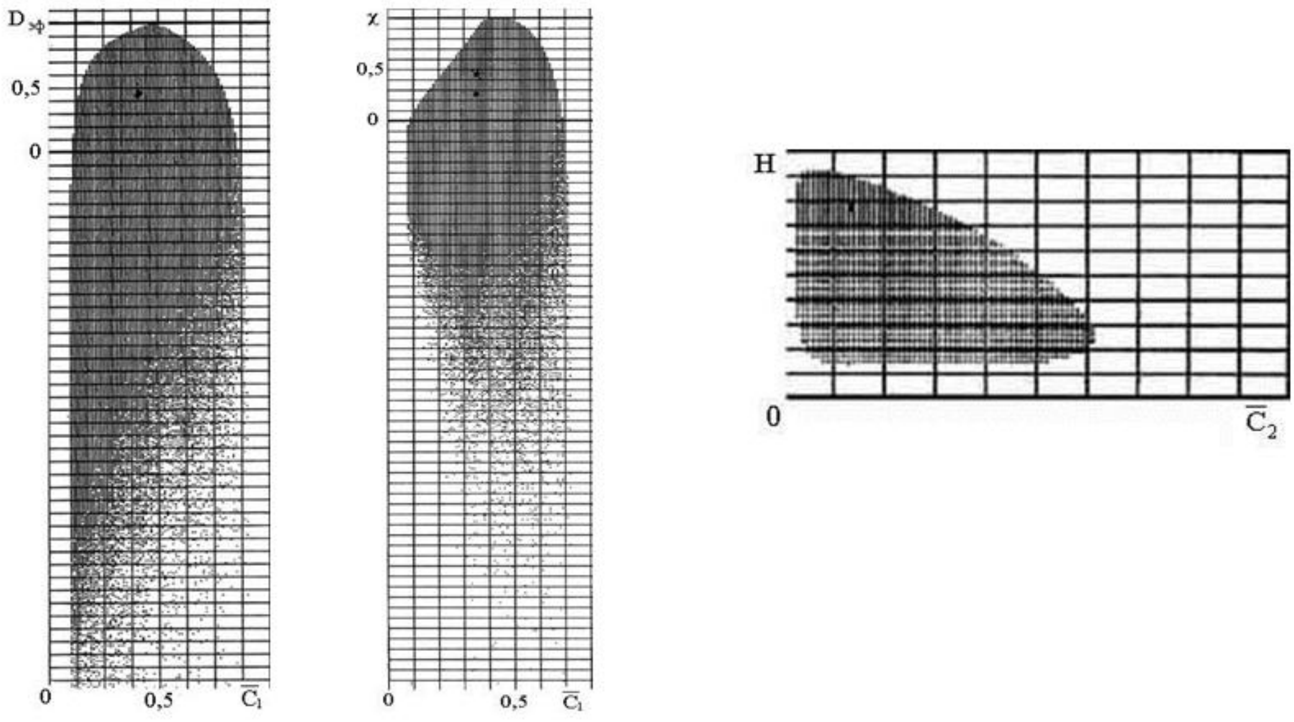


Fig. 4.1. The range of possible values D_{ef} and χ, the range of conditional entropy H allowable values for cluster $\overline{C}_2$.

To determine the management effectiveness a mixed criterion of self-organizing system functioning is proposed:

$$\chi = \frac{H \cdot D_{ef} \cdot \eta^{\Sigma}}{H_0 \cdot D_{ef}^{max}} \qquad (22)$$

Criteria H, D_{ef} и η^{Σ} make it possible to determine the necessary and sufficient conditions of optimal resource distribution in economic systems in a static state (Table 2).

Table 2: Necessary and sufficient conditions of economic system optimal state

Necessary conditions of economic system optimal state	Sufficient conditions of economic system optimal state	Notes
$H \to H_0$ or $H^* = \lvert H - H_0 \rvert \to 0$	$D_{ef} \to D_{ef}^{max}$	There is complete financial information on the economic system (accurate estimate)
$H \to H_0$ or $H^* \to 0$	$\eta^{\Sigma} \to 1$	There is no complete financial information on the economic system (rough estimate)

5. Analysis of the state stability and of a complex system transformation

Using the results and the basic provisions of structure thermodynamics [2] and of nonlinear nonequilibrium fluctuation-dissipation thermodynamics [15], the stability of complex self-developing systems is analyzed.

We consider clusters $\{\overline{C}_i\}$ and subclusters $\{\overline{C}_{ij}\}$, components of the FCC, as random internal parameters C_i (t), C_{ij} (t), changing in a fluctuation manner. If the system is isolated, the information entropy H ($\{\overline{C}_{ij}\}$) does not decrease with time. However, as shown in [16], the fluctuation component of the information (conditional) entropy H ($\overline{C}_{ij}$ (t)) may decrease by an amount not exceeding k (k - Boltzmann constant).

$$\sqrt{\left(\delta H\left(\overline{C}_{ij}(t)\right)\right)^2} < k \tag{23}$$

Expression (23) represents micro infraction of II law of thermodynamics for the FCC.

The subclusters average values at some interval of time τ are:

$$\sum_{j=1}^{5}\left\langle \overline{C}_{ij} \right\rangle = \left\langle \sum_{j=1}^{5} \overline{C}_{ij} \right\rangle = \int_0^{\tau} \overline{C}_{ij}(t)dt \Big/ \tau \tag{24}$$

The conditional entropy H (A (t)) FCC in the case of asymmetric FCM is the following:

$$H(A(t)) = \left\langle \overline{C}_1 + \sum_{j=2}^{5} \overline{C}_{ij} \right\rangle = \left\langle \overline{C}_1 \right\rangle + \sum_{j=2}^{5} \left\langle \overline{C}_{ij} \right\rangle, \tag{25}$$

where A (t) - average internal parameters, $\langle\ \rangle$ - sign of averaging over a certain interval of time τ which is much less than evolution time T from the initial state $\left\{\overline{C}_{ij}^0\right\}$ to the final (ideal) system state $\left\{\overline{C}_{ij}\right\}^{fin(ideal)}$:

$$\tau << T \qquad \left\langle \overline{C}_{ij}^0 \right\rangle \Rightarrow \left\langle \overline{C}_{ij} \right\rangle^{fin(ideal)} \tag{26}$$

In the symmetric case subclusters $\left\langle \overline{C}_{ij} \right\rangle$ are determined by the correlations:

$$\left\langle \overline{C}_{ij} \right\rangle = \left\langle \overline{C}_{ji} \right\rangle \text{ и } \left\langle \overline{C}_{ij} \right\rangle = \left\langle \overline{C}_{i} \right\rangle \cdot \left\langle \overline{C}_{j} \right\rangle, \text{ i.e. } \left\langle \overline{C}_{ij} \right\rangle = \left\langle C_{i} \right\rangle^{2} \tag{27}$$

The conditional entropy in this case is:

$$H = 2\left\langle \overline{C}_{1} \right\rangle - \left\langle \overline{C}_{1} \right\rangle^{2} \tag{28}$$

In accordance with the criterion of thermodynamic stability [17] the second differential of the conditional entropy H for the symmetric case is defined in the following way:

$$\delta^{2}H = \frac{\partial^{2}H}{\partial\left\langle \overline{C}_{1} \right\rangle^{2}}\left(\delta\left\langle \overline{C}_{1} \right\rangle\right)^{2} = -2\left(\delta\left\langle \overline{C}_{1} \right\rangle\right)^{2} \le 0 \tag{29}$$

Thus, for states, close to thermodynamic equilibrium in the symmetric case FCM, the second differential of the entropy $\delta^{2}H$ is negative, i.e., the FCC is stable. The loss of stability limit for symmetric FCM is realized only when $\delta\left\langle \overline{C}_{1} \right\rangle = 0$, that is, at the complete absence of fluctuations in energy cluster $\left\langle \overline{C}_{1} \right\rangle$.

In all other cases at symmetric FCM in states close to the branch of thermodynamic equilibrium the stability criterion is satisfied: $\delta^{2}H < 0$.

In the case of asymmetrical FCM the second differential of the conditional entropy has the form:

$$\begin{aligned} \delta^{2}H\left(\left\langle \overline{C}_{1} \right\rangle, \left\langle \overline{C}_{21} \right\rangle, \left\langle \overline{C}_{22} \right\rangle, \left\langle \overline{C}_{23} \right\rangle, \left\langle \overline{C}_{24} \right\rangle\right) &= \frac{\partial^{2}H}{\partial\left\langle \overline{C}_{1} \right\rangle^{2}}\left(\delta\overline{C}_{1}\right)^{2} + \sum_{j=2}^{5}\frac{\partial^{2}H}{\partial C_{j1}^{2}}\left(\delta\left\langle \overline{C}_{j1} \right\rangle\right)^{2} + \\ &+ 2\frac{\partial}{\partial\left\langle \overline{C}_{1} \right\rangle}\sum_{j=2}^{5}\left(\frac{\partial H}{\partial\left\langle \overline{C}_{j1} \right\rangle} \cdot \delta\left\langle \overline{C}_{j1} \right\rangle\right)\left(\delta\left\langle \overline{C}_{1} \right\rangle\right) + 2\sum_{i=2}^{5}\sum_{j>i}^{5}\frac{\partial^{2}H}{\partial\left\langle \overline{C}_{i1} \right\rangle\delta\left\langle \overline{C}_{j1} \right\rangle}\delta\left\langle \overline{C}_{i1} \right\rangle\delta\left\langle \overline{C}_{j1} \right\rangle \end{aligned} \tag{30}$$

In the case of independence of energy cluster $\left\langle \overline{C}_{1} \right\rangle$ and energy subclusters $\left\langle \overline{C}_{12} \right\rangle$, $\left\langle \overline{C}_{13} \right\rangle$, $\left\langle \overline{C}_{14} \right\rangle$ and $\left\langle \overline{C}_{15} \right\rangle$ the second differential of the conditional entropy $\delta^{2}H$ is determined as follows:

$$\delta^{2}H = 0, \tag{31}$$

that is, even in the presence of fluctuations neutral stability of the complex system evolution occurs.

In the case of linear dependence $\langle\overline{C}_1\rangle$ and energy clusters $\langle\overline{C}_{12}\rangle$, $\langle\overline{C}_{13}\rangle$, $\langle\overline{C}_{14}\rangle$, $\langle\overline{C}_{15}\rangle$ neutral stability also occurs.

In the case of nonlinear dependence of subclusters $\{\langle\overline{C}_{ij}\rangle\}$ (i>1) from the energy cluster there may appear both stable and unstable regimes of the fractal-cluster matrix FCM evolution, i.e.:

$$\delta^2 H \begin{cases} < 0 & - \text{ stable regime} \\ = 0 & - \textit{ neutral stability} \\ > 0 & - \text{ unstable regime} \end{cases} \tag{32}$$

The analysis of a complex system structural stability given above which is based on generalized thermodynamics of irreversible processes by I. Prigogine [2] and the proposed fractal-cluster theory applies to the states, close to thermodynamic equilibrium branch, that is, to the linear thermodynamics of irreversible processes.

The criterion of stability for complex systems, relevant to concept of "dissipative structures" by I. Prigogine, is a quadratic alternating form, called the increment of entropy production [2]. For the stable dissipative structures the excess entropy production is a positively definite value:

$$P[\delta H] > 0, \tag{33}$$

where $\delta H = \frac{\partial H}{\partial\langle\overline{C}_1\rangle}\delta\langle\overline{C}_1\rangle + \sum_{j=2}^{5}\frac{\partial H}{\partial\langle K_{1j}\rangle}\delta\langle\overline{C}_{1j}\rangle$.

As noted in [3], the sign of excess entropy production in general can not be determined definitely. For specific systems the use of phenomenological laws for determining the sign is required $P[\delta H]$.

For the fractal-cluster description of the ES structure, located far from equilibrium, the following expression for the quadratic alternating forms has been got, i.e., for the production of excess entropy (or quasientropy) for the symmetric case FCM $(\overline{C}_{ij} = \overline{C}_{ji})$:

$$P[\delta H]=\begin{cases}\delta\langle\overline{C}_1\rangle<\dfrac{B}{1-\langle\overline{C}_1\rangle}, B>0 & -\ \text{stable regime}\\ \delta\langle\overline{C}_1\rangle=\dfrac{B}{1-\langle\overline{C}_1\rangle} & -\ \text{neutral regive}\\ \delta\langle\overline{C}_1\rangle>\dfrac{B}{1-\langle\overline{C}_1\rangle} & -\ \textit{unstable regime}\\ H<H_0 & \end{cases} \quad (34)$$

where B is determined by the initial values $\langle\overline{C}_1\rangle^0$ и $\delta\langle\overline{C}_1\rangle^0$.

Thus, we can conclude that the synthesis of fractal-cluster theory and generalized thermodynamics of irreversible processes allows an explicit to determine the type of stability criterion far from ES equilibrium state.

Let us consider the problem of stability of transition trajectory of a complex system from arbitrary in ideal condition in accordance with the fractal-cluster theory basic provisions. Obviously, both stable and unstable trajectories of system transformation can go through the two points in the phase plane entropy - time (H - t) in terms of fractal-cluster theory.

Let us consider a fractal-cluster structure of a complex system in a state close to thermodynamic equilibrium branch, that is, while analyzing evolution it is possible to use the linear thermodynamics of nonequilibrium processes.

In accordance with this fact, we can use the theorem of minimum entropy production [2].

For simplicity we consider a symmetric fractal-cluster matrix (FCM) of a topological structure, then the entropy of a complex system is determined by (31).

Using I. Prigogine's theorem of the minimum entropy production [2], we define the function $f(\bar{t})$ from the condition of neutral stability:

$$\frac{dP}{d\bar{t}}=0\text{, where } P=\frac{dH}{d\bar{t}} \quad (35)$$

Expression of transformation function $f(\bar{t})$, which executes transition trajectory of a complex system fractal-cluster structure from arbitrary into ideal state, in the trajectory of neutral stability is the following:

$$f(\bar{t})=\{\exp(\alpha)-1\}^{-1}(\exp[\alpha\cdot\bar{t}]-1) \quad (36)$$

In general case of non-zero right-hand side in the expression for entropy production we obtain the following expression for the transformation function $f(\bar{t})$:

$$f\left(\bar{t},\varepsilon\right)=-\frac{\varepsilon}{\alpha}\bar{t}+\left(1+\frac{\varepsilon}{\alpha}\right)\cdot\left(\exp\left(\alpha\right)-1\right)^{-1}\left(\exp\left(\alpha\cdot\bar{t}\right)-1\right) \quad (37)$$

The expression for the function $f\left(\bar{t},\varepsilon\right)$ conforms to the following qualitatively different transformation modes of the topological fractal-cluster structure of a complex system from non-ideal in an ideal state:

$$\varepsilon = \begin{cases} = 0 \text{ – transformation of a complex system by the trajectory of a neutral stability} \\ > 0 \text{ – unsustainable trajectory of a complex system transformation} \\ < 0 \text{ – steady trajectory of a complex system transformation} \end{cases} \quad (38)$$

6. Probation FCC - theory for the EC of micro-, meso- and macro-level

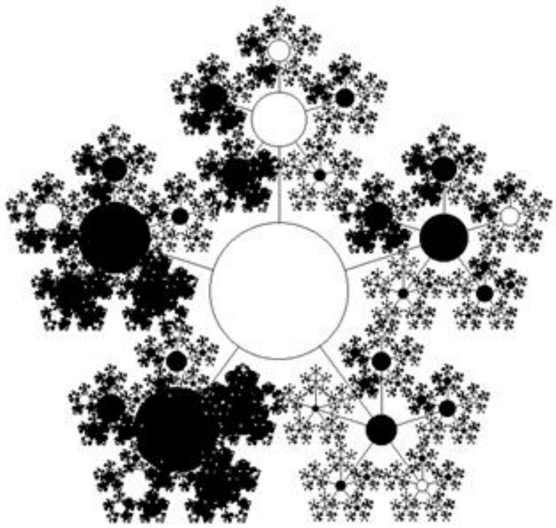
A retrospective fractal-cluster analysis of municipal structures management for the Moscow region and the Municipal Department of Nashua, USA is presented in Table 3 as the first example. As Table shows, for the U.S. Municipal Department FCC is almost perfect, the criterion of management effectiveness D_{eff} and the total system efficiency is close to 100%. For the municipal structures of the Moscow region the most successful in terms of governance is the year of 1993.

The generalized criterion χ for Nashua city, USA is maximum, which indicates optimal management .

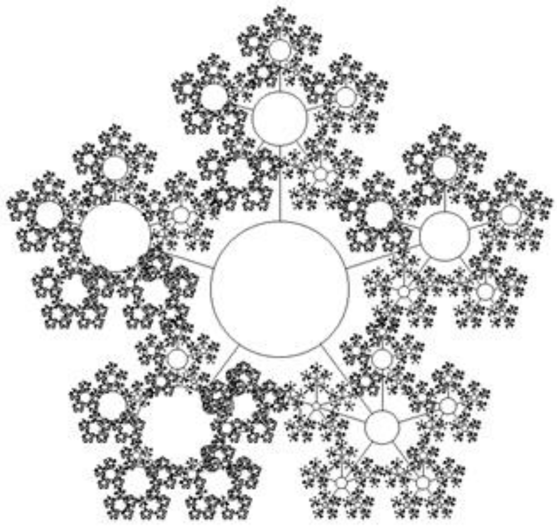
Table 3. Comparative analysis of municipal structures management

Structure name	Entropy		The efficiency criterion D_{eff}	Full efficiency η^{Σ}	Relative deviation from ideal		
					ε_H	ε_D	ε_η
Municipal structure of the Moscow region	1990	0,360	0,1132	0,83	41,7%	88,6%	17%
	1993	0,564	0,8755	0,969	8,7%	12,5%	3,1%
	1996	0,407	0,7227	0,9257	3,4%	27,8%	7,5%
Municipal Department of Nashua city, USA	1993	0,603	0,97	0,99	2,42%	3%	1%
	1994	0,6156	0,9875	0,9957	0,4%	1,2%	0,4%

Figure 6.1 shows the various scenarios of FCM topological structures for different time stages of FCM evolution to its ideal value.



t = 0,5; φ = 72°



t = 1; φ = 72°

Fig. 6.1. Evolution of FCC sixth level φ = 72 º

Fig. 6.2. Evolution of FCC sixth level t = 0 φ = 20º

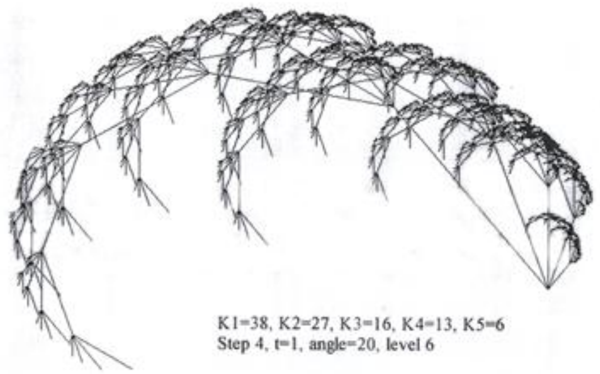
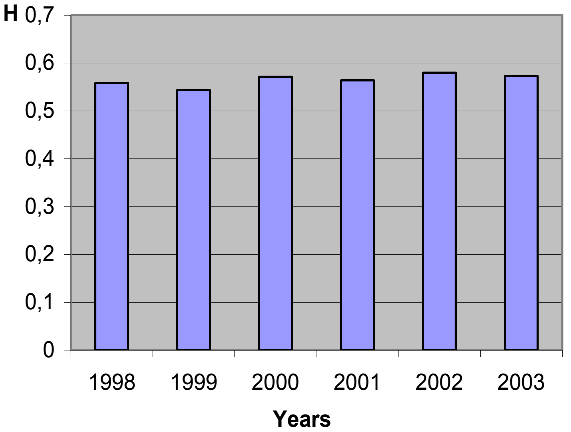
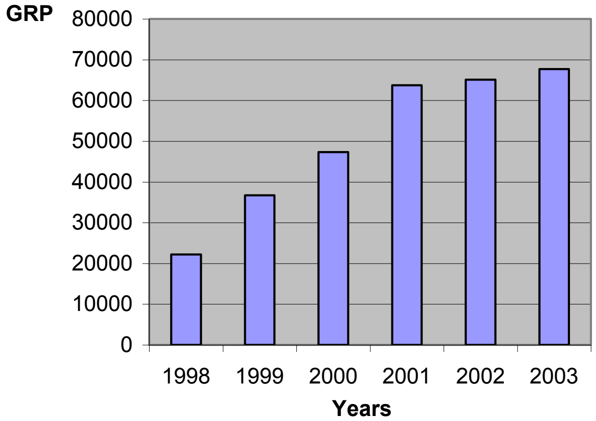


Fig.6.3. Evolution of FCC sixth level φ = 20º

Fractal-cluster analysis of the Samara region budget on the data of 1995-2001 is presented in histograms (Fig. 6.4, 6.5). From these illustrations it is seen that there is a weak oscillation of fractal cluster entropy around the value of $H_0 \approx 0{,}618$ – “golden section” $\left(\Delta H / H_0 \approx 0{,}01 \div 0{,}03\right)$. This proves the normal distribution of budget resources in the region from 1995 to 2001. The results of fractal-cluster analysis are supported by statistical data on the GRP, the pace of economic growth, investment, rising of living standards in Samara region (Samara region is one of the three most economically successful regions of Russia).



a)



b)

Fig. 6.4. Histogram of the Samara region entropy budget

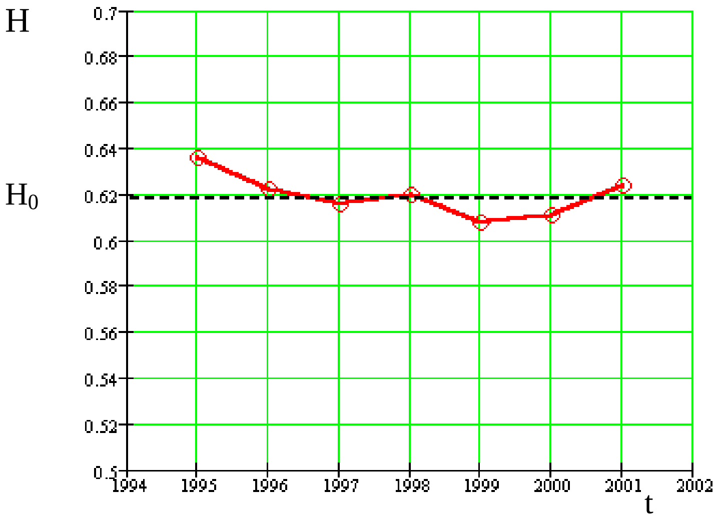
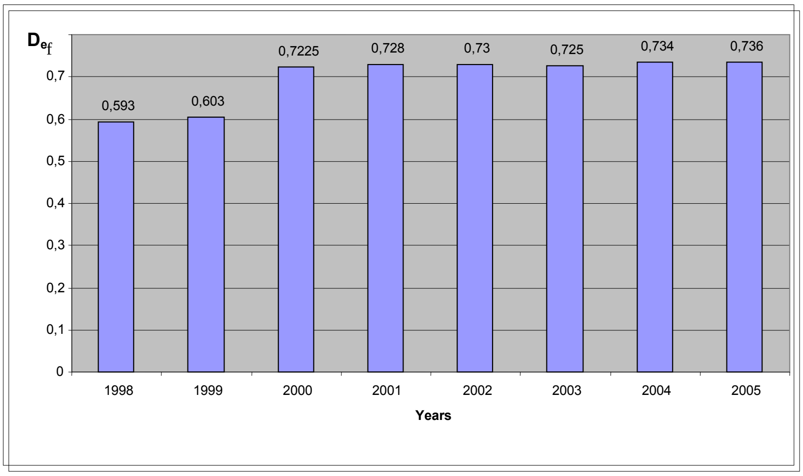


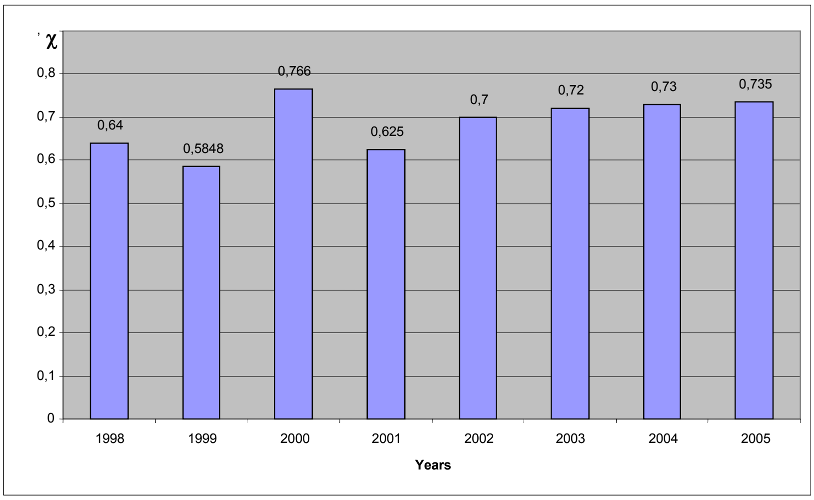
Fig. 6.5. Changes in FC - entropy of the budget structure for the period 1995-2001 in Samara region

As seen from Fig. 6.4. and 6.5. there is a confirmation of the hypothesis of low intensity waves H (functional instability) for the successful development of ES.

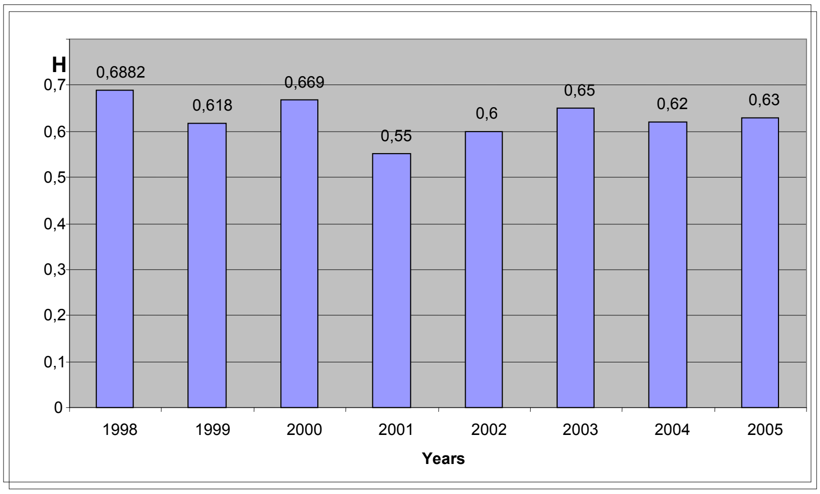
One more example illustrating the developed theory of resource distribution in micro level ES is the results of analysis for Samara State University of Railway Engineering.



a)



b)



c)

Fig.6.8. Histograms of the distribution D_{ef} - criterion (a), the generalized criterion χ (b), and the information entropy (c) in the financial structure of Samara State University of Railway Engineering budget from 1998 to 2005.

Figure 6.8. shows the histogram of criterion of effective university management D_{ef} (a), the generalized criterion χ (b) and information entropy of the topological structure of the university budget H for Samara State University of Railway Engineering in the period from 1998 to 2005.

These histograms show clearly that the characteristics of academy management (N = 0,688; D_{ef} = 0,593; χ = 0.64) were the worst in 1998, because of the general economic situation of that time in the country; the consequences of default are noticeable in the next 1999, respectively (H = 0.618; D_{ef} = 0.603, and χ = 0.5848).

It should be noted that despite the fact that the information entropy distribution of the budget topological structure is close to the "golden section" (H_0= 0.618), other criteria D_{ef} and χ are not equal to their maximum values ($D^{max} = \chi^{max}$ =1), i.e. there is an imbalance in the structure of the academy budget. This fact ($H=H_0$) can be explained by the following: with scarce university budget and high living wage (1250 rubles per month in 1999) university authorities did not redistribute consolidated budget to increase wages, which could lead to entropy H increase.

In terms of the structural budget management the best in the academy is the year of 2000 (H = 0.669; D_{ef} = 0.722; χ = 0.766). The deterioration of consolidated budget management structure in 2001 is driven by objective reasons:

1) reduction of wage supplements from the Ministry of Transport, which led to a decrease in the energy cluster, and thus to the entropy H decrease;
2) reconstruction of the university (construction of buildings, dormitories), modernization and equipping of the laboratory-technical base, financed by the Ministry of Transport and Kuibyshev Railway led to a relative increase of technological cluster.

However, as the analysis shows, this endogenous components fluctuation as parts of the university budget has a slight and transitory nature.

Important indicators of stability in the academy development are a high integrated indicator of quality - demand for graduates ~ 96% (in Russia on average the figure is 50%) and high rates of the university employees' salary growth.

7. A generalized fractal-cluster criterion of ES management optimizing

The result of information and thermodynamic analysis of recourse distribution in economic systems based on fractal-cluster model makes it possible to formulate a new generalized criterion to optimize the economic systems management - optimal control of the economic system in terms of fractal-cluster model - this is, in contrast to traditional notions (minimum cost or maximum profit), the sustainable development of the crisis-free economic system, which corresponds to extreme static fractal-cluster criteria ($D_{ef,}$ η^{τ},H) and obtained solutions for sustainable transformation (dynamic stability criteria).

This generalized criterion of management optimization is a combination of static and dynamic fractal-cluster criteria (D_{ef}, H, d^2H/dt^2, χ, δ^2H, P[δH]), and decisions on sustainable transforming the micro-and meso level economic system (13 - 16, 37), i.e., represents the conditions for sustainable crisis-free operation of the ES. However, this criterion does not reject the traditional criteria - the maximum profit and minimum cost, but enables the synergetic solution to problems of ES management optimization.

8. Conclusion

The developed fractal-cluster theory allows to analyze and optimize the ES in the aspect of resource distribution. A new generalized criterion of the ES management optimization is formulated on the basis of fractal-cluster criteria and solutions for sustainable transformation that have been developed. Testing of the developed theory of analyzing resource distribution in the micro-and mesolevel ES affirmed its foundations and recommendations.

Fractal-cluster theory and models worked out on its basis will be dominant in the prognosis of ES development, where it is impossible or difficult to evaluate the system product in terms of value (educational institutions, basic research, etc.). For the ES «inputs - output", with a high degree of probability while predicting the products and assessing the ES effectiveness (profitability, sales, GDP, etc.), traditional economic and mathematical models will dominate. For this class of ES the proposed fractal-cluster theory and models based on it can be used as auxiliary tools of ES management analysis.

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